Incomplete Tax Enforcement, Managerial Quality and Economic Development *

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Abstract

Using establishment-level World Bank Enterprise Surveys, I document the following trends: (i) the average tax noncompliance rate, defined as the ratio of unreported sales to total sales, decreases with GDP per worker, (ii) the tax noncompliance rate is size-dependent, i.e., small establishments conceal a higher fraction of their sales than large establishments, (iii) the level of this size-dependency diminishes as GDP per worker increases. To examine the implications of these findings for managerial quality and aggregate output, I develop a modified version of Lucas (1978) span-of-control model in which managers invest in their managerial skills and choose how much of their income to report to the government after considering the risk of getting inspected by tax officials. The results reveal that incomplete tax enforcement significantly diminishes economy-wide managerial quality, with the magnitude of this impact escalating with the level of size-dependency in tax noncompliance. For instance, transitioning from the benchmark economy, calibrated to U.S. data, to an economy similar to Brazil's tax enforcement regime leads to an approximate 23% reduction in average managerial quality and roughly a 3% decrease in output.

Keywords: Tax Enforcement, Managers, Skill Investment, Cross-Country Income Differences

JEL Codes: E23, E26, J24, L26, O17

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1 Introduction

It's widely recognized that firms in developed countries are more productive compared to those in developing countries, indicating their ability to achieve greater output from a given set of inputs. This capability, extensively explored in the literature, can be attributed to disparities in several factors, including human capital, innovation, competition, or institutional arrangements across countries.[1] Furthermore, a relatively recent body of literature has emphasized the pivotal role of managers in cross-country productivity differences. Bloom et al. (2012) highlight that management practices exhibit significant variations across countries, i.e., the worst management practices are often found in developing countries. Additionally, Guner et al. (2018) document that managers in less developed countries accumulate less human capital throughout their careers compared to their counterparts in developed countries.

Another salient feature of developing countries is that their governments typically struggle with inadequate tax collection abilities and monitoring compliance with the tax code. The tax gap, defined as the difference between tax revenues that should be collected by law and the amount actually collected, can reach up to 6 percent of GDP in developing countries.[2] In this paper, I study the relationship between incomplete tax enforcement, managerial quality, and economic development. Broadly speaking, the connection between incomplete tax enforcement and managerial quality can be attributed to multiple factors. For example, individuals with lower talent might be more motivated to become managers in countries with lenient tax enforcement due to the tax noncompliance opportunities. Alternatively, managers might be discouraged from enhancing their skills if tax administration favors lower-ability managers over those with higher abilities.

To begin with, I provide evidence regarding the tax noncompliance rate, which I define as the fraction of output (sales) not reported to tax authorities for tax purposes, and its correlation with GDP per worker and managerial quality. Using data from the World Bank Enterprise Surveys (WBES), I initially demonstrate that the average tax noncompliance rate diminishes as GDP per worker increases. In other words, establishments in less developed countries, on average, declare a smaller fraction of their sales to tax authorities compared to those in developed countries. For instance, the average tax noncompliance rate in Germany

[1] See for example: Benhabib and Spiegel (1994), Erosa et al. (2010), Gennaioli et al. (2013) for human capital and productivity; Griffith et al. (2004), Jorgenson et al. (2008) for innovation and productivity; Melitz (2003), De Loecker (2011) for competition and productivity; Hsieh and Klenow (2009),Acemoglu et al. (2004), Hopenhayn and Rogerson (1993) for institutions and productivity

[2] For example: 2% in South Africa, 3.3 % in Turkey, and 5.9% in Mexico. See Jansen et al. (2020) and Khwaja and Iyer (2014)

and Spain hovers around 4-5%, whereas it exceeds 25% in countries like Turkey, India, and Mexico.

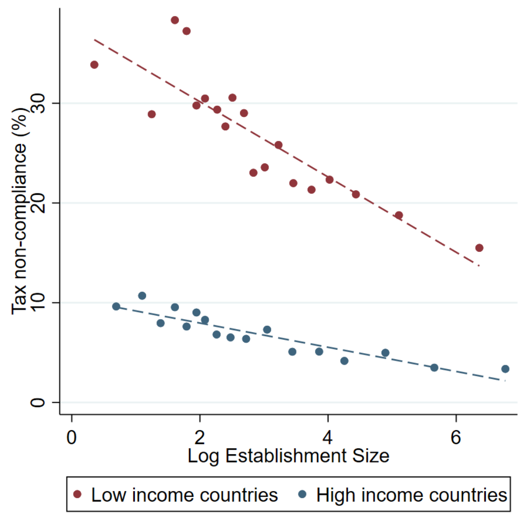
Next, I proceed to show that the tax noncompliance rate is size-dependent. To be more precise, within a given country, smaller establishments tend to conceal a larger portion of their sales from the government compared to larger establishments. This pattern persists even after accounting for various potential confounding factors, including establishment age, industry, location, and export status. The negative correlation between tax noncompliance and establishment size remains consistent across nearly all countries in the sample.

Furthermore, I provide evidence indicating that the degree of size-dependency diminishes as GDP per worker rises. Put differently, the disparity in the average tax noncompliance rate between small and large firms is less pronounced in wealthier countries compared to poorer countries. For instance, in Germany, a one-log point increase in establishment size corresponds to approximately a one-percentage-point decrease in the tax noncompliance rate, whereas in Mexico, this effect is much higher, with the number being as large as four. Additionally, taking into account the observation that higher GDP per worker aligns with enhanced management practices and the accumulation of human capital among managers throughout their careers, a significant association also emerges between the degree of size-dependency in tax noncompliance and the quality of management.

Figure 1 provides a snapshot of these empirical findings regarding tax enforcement across countries. It consists of a binned scatterplot depicting tax noncompliance against the logarithm of establishment size for both high-income and low-income countries. This correlation is unconditional. On average, tax noncompliance is higher in lower-income countries. The relationship between tax noncompliance and size-dependency is consistently negative, regardless of whether the countries are low or high income. However, the slope of this relationship is steeper in low-income countries compared to high-income countries. This implies that a one-unit increase in establishment size is associated with a greater reduction in tax noncompliance in poorer countries than in wealthier ones.

Based on the insights obtained from these empirical findings, I pose a question: Could the differences in how countries enforce taxes be a reason for the varying quality of management and income levels between countries? To explore this, I develop a modified version of Lucas (1978) span-of-control model in which managers invest in their managerial skills and choose how much of their sales to report to the government after considering the risk of getting inspected by tax officials. In this model environment, individuals decide to be either workers or managers at the very beginning of their lives. The current ability of a manager determines the size of the establishment that she operates. This model setup allows the coexistence of

Figure 1: Tax Noncompliance and Establishment Size



Sources: World Bank Enterprise Surveys and author's calculations. **Notes:** The figure consists of binned scatterplots illustrating the relationship between the tax non-compliance rate and the logarithm of establishment size for firms in both low-income and high-income countries, classified according to the World Bank's income categories. The binscatter method divides the log establishment size variable into equal-sized bins. Within each bin, it calculates the mean values of both log establishment size and tax noncompliance, and then presents these data points in a scatterplot. In essence, each point in this figure represents a bin of establishments. For a more detailed version of this figure, please refer to the Figure A1

establishments with different sizes and tax noncompliance rates. Inspired by the fact that tax noncompliance is size-dependent, I make the assumption that the likelihood of being inspected increases with the manager's ability. This means that the chance of being inspected is higher for larger establishments.[3] Under this assumption, high-quality managers choose to comply more with the tax code; thus, large establishments face higher effective tax rates. This formulation introduces a mechanism where incomplete tax enforcement influences both managerial quality and overall output through two primary channels.

The first channel, referred to as the *occupation channel*, operates through individual occupation decisions. Notably, the option to avoid complying with the tax code makes the role of a manager more appealing. This leads to certain individuals with lower abilities, who would otherwise prefer to be workers in a scenario with full tax enforcement, choosing to become managers. This influx of lower-ability individuals into managerial roles adversely affects the overall average quality of management.The second channel, referred to as the *skill investment channel*, emerges from decisions regarding investment in managerial skills. As observed by Guner et al. (2018), size-dependent distortions have a negative impact on the investment in managerial skills. Similarly, in the framework I propose, the size-dependent

[3] This modeling choice is in line with Di Nola et al. (2021), Kuehn (2014) and Leal Ordóñez (2014).

nature of tax noncompliance acts as a disincentive for managers to invest in enhancing their skills. This is because they are aware that operating a larger establishment increases the likelihood of being subjected to audits.

I calibrate the benchmark model to the U.S. economy under the assumption that the tax code can be completely enforced. The benchmark model successfully mimics central features of U.S plant size distribution and managers' age-earning profiles.[4] Subsequently, I conduct a set of quantitative exercises to understand the role of size-dependency of tax enforcement in both managerial quality and output. The main finding of these exercises is that size-dependent tax enforcement has a substantial and negative effect on managerial quality and output. In addition, the magnitude of the effect gets larger as the level of size-dependency in tax noncompliance increases.

To bridge the model framework with empirical observations, I examine scenarios where the benchmark economy is compared to economies where the probability of getting inspected parameters is calibrated to match the tax noncompliance patterns observed in less developed countries. This means that while keeping all other parameters constant, I calibrate the parameters governing tax noncompliance to match tax noncompliance related moments observed in other countries. The results of this exercise reveal, for instance, that transitioning from the benchmark economy to an economy similar to Brazil's tax enforcement regime leads to an approximate 23% reduction in average managerial quality and roughly a 3% decrease in output. Notably, the design of this hypothetical Brazilian economy implies that these differences between the two economies are exclusively attributable to the size-dependency of tax enforcement.

My findings indicate that younger and highly skilled managers are significantly more affected by these distortions compared to their older and less skilled counterparts. Additionally, I perform a decomposition exercise to gain insight into the relative importance of the *skill investment channel* and the *occupation channel*. The results suggest that both channels are of great quantitative importance in determining average managerial quality in the economy, while the *skill investment channel* is the primary determinant of the effects on output. The adjustments in prices in the general equilibrium help alleviate the decline in aggregate output resulting from the deterioration in managerial quality.

The remaining of the paper is organized as follows. Section 2 examines the related literature briefly. Section 3 describes data and documents relevant facts about tax noncompliance. Section 4 presents the model, while Section 5 describes the calibration procedure. Section 6 present the results of quantitative exercises. Section 7 analyzes quantitative findings. Section

[4] I refer to a production unit as a plant. I use the word "establishment" and "plant" interchangeably.

8 provides a brief road map for future research and concludes.

2 Related Literature

This paper contributes to three strands of literature. First, it aligns with recent research that focuses on resource misallocation as a source of cross-country productivity differences. Previous studies like Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) have measured the macroeconomic effects of resource misallocation across firms without explicitly identifying its source. Many studies in this literature pinpoint potential sources of misallocation and quantify their effects, including credit market imperfections (Banerjee and Duflo (2005), Gopinath et al. (2015), Midrigan and Xu (2014)), size-dependent policies (Guner et al. (2008)), firing taxes (Hopenhayn and Rogerson (1993)), trade barriers (Eaton et al. (2011), Pavcnik (2002), Caliendo and Parro (2014)), and poor contract enforcement (Caselli and Gennaioli (2013)). Similarly, this paper documents differences in tax enforcement across countries and considers it a potential source of factor misallocation while quantifying its effects.

Therefore, naturally, this paper is closely linked to the literature on tax enforcement. This literature traces its origins back to the tax deterrence model proposed by Allingham and Sandmo (1972). Since then, economists have delved into various questions about incomplete tax enforcement over the years, such as determining the factors influencing tax noncompliance (as explored by Dabla-Norris et al. (2008) and Alm (2012)), defining the optimal tax enforcement policies (as discussed by Keen and Slemrod (2017) and Bigio and Zilberman (2011)), and investigating the macroeconomic consequences of incomplete tax enforcement (as studied by Di Nola et al. (2021), Fernández-Bastidas (2023), Asatryan and Peichl (2017)). Among these, two papers closely align with this one in the sense that they combine tax enforcement and misallocation literature: Leal Ordóñez (2014) and Bachas et al. (2019).

Leal Ordóñez (2014) assessed the quantitative effect of incomplete tax enforcement on aggregate output and productivity. However, there are two critical differences between my work and Leal's study. Firstly, he investigated informality at the extensive margin, categorizing establishments as either formal or informal. In contrast, my study goes beyond this duality by characterizing informality at the intensive margin. This margin may be potentially quantitatively important, especially when considering that, although all the establishments in the WBES dataset are registered (formal) firms, nearly half of them report positive tax noncompliance. Second, in contrast to Leal Ordóñez (2014), managerial quality is endogenous in my study. Furthermore, I establish a connection between incomplete tax enforcement and low managerial quality, both of which are common phenomena in developing countries. Bhattacharya et al. (2013) demonstrates that the distortion of size-dependent policies significantly impacts managerial skill investment, underscoring its quantitative importance.

Bachas et al. (2019) employ rigorous empirical methods to establish a causal relationship between an industry's average firm size and its tax inspection probability and compliance rate. They found a positive gradient, with the magnitude most pronounced in the poorest countries and nonexistent in wealthier nations. They subsequently integrate these empirical findings into their model, which includes three channels through which size-dependent tax noncompliance can impact total factor productivity (TFP): resource misallocation among existing firms, the entry and exit dynamics of firms, and incentives for innovation. The mechanism of innovation incentives in their study bears similarity to the managerial skill investment mechanism discussed in this paper. While the innovation incentive mechanism in Bachas et al. (2019) connects tax noncompliance to lower productivity and flatter growth trajectories of firms in less developed countries, the managerial skill investment mechanism in this paper highlights a connection between incomplete tax enforcement and poorer managerial practices, as well as flatter income growth trajectories for managers in less developed countries. This links my paper to another influential literature on managerial quality and development.

Recent literature has revealed significant variations in management practices across countries, with better management practices being linked to higher productivity. Bloom et al. (2012) have attributed these cross-country differences in management practices to factors like product market competition and labor market regulations. Additionally, some studies have emphasized the role of individual managers in plant efficiency. For instance, research by Bushnell and Wolfram (2009) found that power plant operators have a substantial impact on the thermal efficiency of power plants. Similarly, Bertrand and Schoar (2003) examined the effects of top executives (such as CEOs and CFOs) and found that these executives had a significant explanatory power over firms' returns on assets. Furthermore, studies by Bhattacharya et al. (2013) and Guner et al. (2018) have highlighted size-dependent distortions, which are more prevalent in less developed countries, leading to lower human capital accumulation among managers in these nations. My paper contributes to this discussion by suggesting that incomplete tax enforcement may be a factor contributing to cross-country differences in managerial skills.

3 Empirical Findings

In this section, I provide evidence regarding tax noncompliance at the establishment level across various countries, using data from the World Bank Enterprise Surveys (WBES). The key findings can be summarized as follows: Firstly, there is a consistent trend where the average tax noncompliance rate decreases as GDP per worker increases. Secondly, it's observed that tax noncompliance rates tend to be size-dependent across almost all countries, implying that smaller firms exhibit higher levels of tax noncompliance compared to larger firms. Thirdly, the degree of size-dependency in tax noncompliance decreases as GDP per worker rises. In addition to these, I demonstrate a strong negative correlation between the degree of size-dependency in tax noncompliance and the quality of management practices.

3.1 Data

World Bank Enterprise Survey (WBES) is an establishment-level survey of a representative sampling of an economy's private sector. The surveys offer a broad range of variables related to access to finance, corruption, crime, competition, and performance measures. They span data from over 194,000 firms in 155 countries, covering the period from 2002 up to the present day. In the earlier survey years, respondents are asked the following question: *"Recognizing the difficulties that many firms face in fully complying with taxes and regulations, what percent of total annual sales would you estimate the typical firm in your area of business reports for tax purposes?"*. I employ the response to this question as a proxy measure for tax compliance and calculate the tax noncompliance rate by subtracting this response from 100.[5] The reference to a "typical establishment in your area of activity" is deliberately used to encourage establishments to provide accurate information regarding their own tax-reporting behavior (Bachas et al. (2019)).

This question was posed over a five-year period spanning from 2002 to 2006. I made the decision to exclude government-owned establishments from the sample.[6] The final sample covers 59,366 establishments from 91 countries. Within this final sample, establishments from various income groups are represented. Specifically, it includes establishments from 7 high-income countries, 21 upper-middle-income countries, 42 lower-middle-income countries, and 29 low-income countries.[7] The full list of countries, along with the number of establishments

[5] For further examples of research employing the same variable for this purpose, see for example: Dabla-Norris et al. (2008), Bachas and Jensen (2017), Bachas et al. (2019), and Alm et al. (2019).

[6] Number of government-owned firms in the sample is 4450, which constitutes 7.4 percent of the whole sample.

[7] Eight countries transitioned from one income category to another across different survey years. For

and survey years, can be found in Table A1.

Table 1: Summary Statistics

A. Basic Summary Statistics of Relevant Variables				
Variables	Number of Observation	Response Rate (%)	Mean	Standard Deviation
Establishment Size	57,986	97.7	63.4	125.3
Establishment Age	58,987	99.3	16.6	18.0
Tax Noncompliance	49,749	83.8	21.4	30.5

B. Distribution of Tax Noncompliance					
Noncompliance rate	0 %	1-20 %	20-40 %	40-60 %	>60 %
Share of establishments (%)	51	10	15	9	15

Summary statistics for the relevant variables are presented in Table 1. In Panel A, you can find the mean, standard deviation, and response rates regarding the size, age, and noncompliance behavior of establishments. The size of the establishments refers to the number of full-time and permanent employees. On average, the establishments in the sample have a size of 63.2, but there is significant variation, with a standard deviation of 125.[8] The average age of an establishment is approximately 16 years.

Regarding the tax noncompliance rate, although it's a sensitive topic for a business survey, the response rate is quite high, at almost 84 percent. The mean tax noncompliance rate is 21.4, with a standard deviation of 30.5. In Panel B, you can see that the distribution of tax noncompliance rates across the sample is highly right-skewed. Nearly half of the establishments report no tax noncompliance, while only 15 percent report more than 60 percent noncompliance.

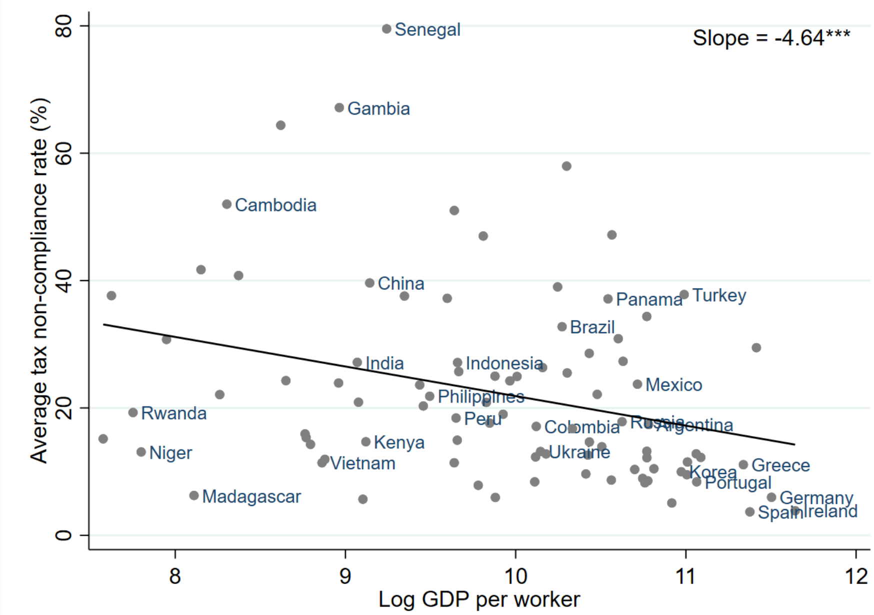
3.2 Stylized Facts

Average tax noncompliance rate decreases with GDP per worker. It is a well-established fact that the size of the informal economy is generally higher in less developed

instance, Turkey was classified as a lower-middle-income country in 2002 but moved up to upper-middle-income status in 2005 according to World Bank classifications.

[8]The average number of employees in the establishments in the sample is 63, which is relatively high compared to country-level business surveys. For example, according to U.S. economic census, the average size in the USA manufacturing sector is around 17 workers, and in many developing countries, it's even less than 10 workers. The underlying reason for this difference is that Enterprise Surveys only include establishments with more than 5 workers.

Figure 2: Average Tax noncompliance and GDP per Worker



Sources: World Bank Enterprise Surveys and World Bank Data. **Notes:** This figure shows the relationship between the average tax noncompliance rate and the log GDP per worker at the cross-country level. Each dot represents a country. The solid line is the simple regression line where the average tax noncompliance is the dependent variable and the GDP per worker is the independent variable. The slope of the regression line is -4.64 and it is statistically significant at the 1 percent level.

countries than in developed countries. I document that not only the size of the informal sector but also the average tax-non compliance rate decreases with GDP per worker.[9] Figure 2 shows the relationship between the average tax noncompliance rate and log real GDP per worker.[10] One log point increase in real GDP per worker is associated with a 4.65 percent decrease in the average tax - noncompliance rate. This relation is statistically significant at the 1% level. The average tax noncompliance of the countries is listed in Table A1 column 4.

Tax noncompliance rate is size-dependent. I document that the tax noncompliance rate is not homogeneous across establishments with different sizes. More specifically, the tax noncompliance rate increases with the establishment size, measured by the number of full-

[9]WBES is an establishment-level survey of a representative sampling of an economy's private sector. To ensure representativeness, the WBES use a stratified random sampling method in which the establishments are stratified by size, sector, geographic region within a country. Therefore, I calculate the average tax noncompliance rate by simply taking an average across the related sample.

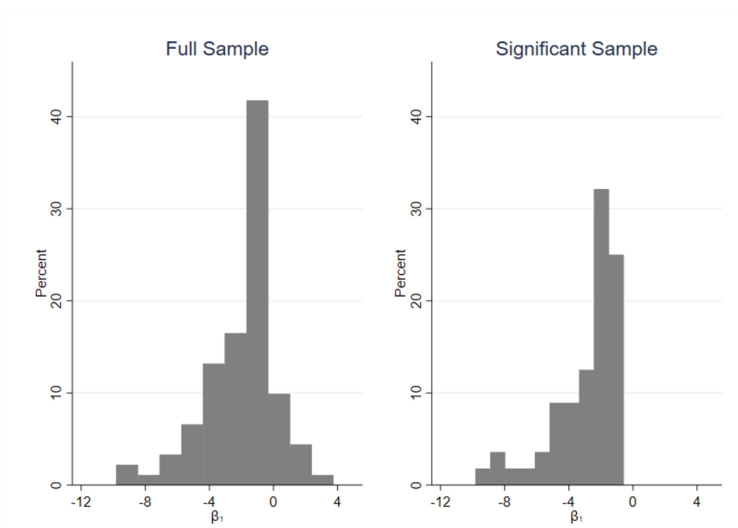
[10]The date of sample collected varies across countries. I constructed GDP per worker measure by taking the average GDP per worker between 2002-2006 for each country.

time workers. To formalize this observation, I use the following simple regression framework:

$$e_i = \beta_0 + \beta_1 logsize_i + \beta_2 controls_i + \epsilon_i \tag{1}$$

where e_i is the tax noncompliance rate of establishment i, and $logsize_i$ is log of the number of full-time permanent workers employed in establishment i, and $controls_i$ is the set of controls, including establishment age, sector, ownership status, and exporter status. The left panel of Figure 3 demonstrates the distribution of size coefficients β_1 regardless of their statistical significance. For most of the countries, namely 81 countries out of 91, estimated size coefficients are negative, which in turn implies that larger establishments comply more with tax code than smaller establishments in these countries. The right panel draws the same distribution by narrowing down the sample to countries with statistically significant size coefficients at 10 percent level.[11] It shows that, without any exception, all of the statistically significant size coefficients are negative.

Figure 3: Distribution of Size Coefficients



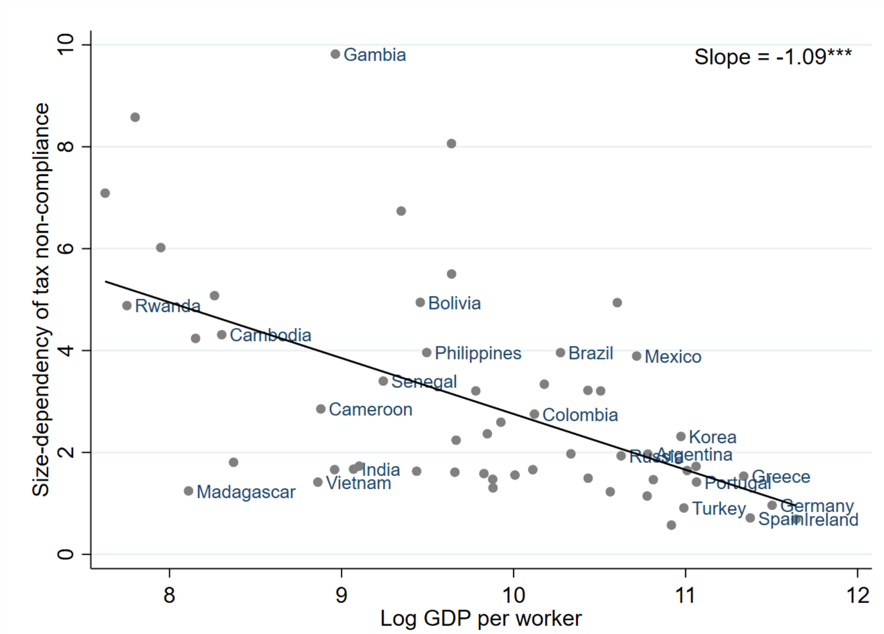
Sources: World Bank Enterprise Surveys, World Bank and author's calculations. **Notes:** The left panel of this figure shows the distribution of size coefficients for the whole sample of countries. Size- coefficients β_1 are estimated from equation 1. The right panel draws the same distribution by narrowing down the sample to countries with statistically significant size coefficients at 10 percent level. The size coefficient estimates are statistically significant at 10 percent level in 56 countries.

The level of size-dependency decreases with GDP per Worker. I defined the level of size-dependency of tax noncompliance, or simply level of size-dependency, as the ab-

[11] The size coefficient estimates are statistically significant at 10 percent level in 56 countries.

solute value of size coefficient, β_1 in equation 1. Thus, by definition, a higher level of size-dependency means a larger difference of tax noncompliance between small and large establishments. Figure 4 illustrates that level of size-dependency of tax noncompliance is higher in richer countries than poorer countries, which in turn implies that the tax non-compliance differences between small and large establishments are higher in less developed countries. Specifically, one log point increase in real GDP per worker is associated with a 1.09 percentage point decrease in the level of size-dependency.

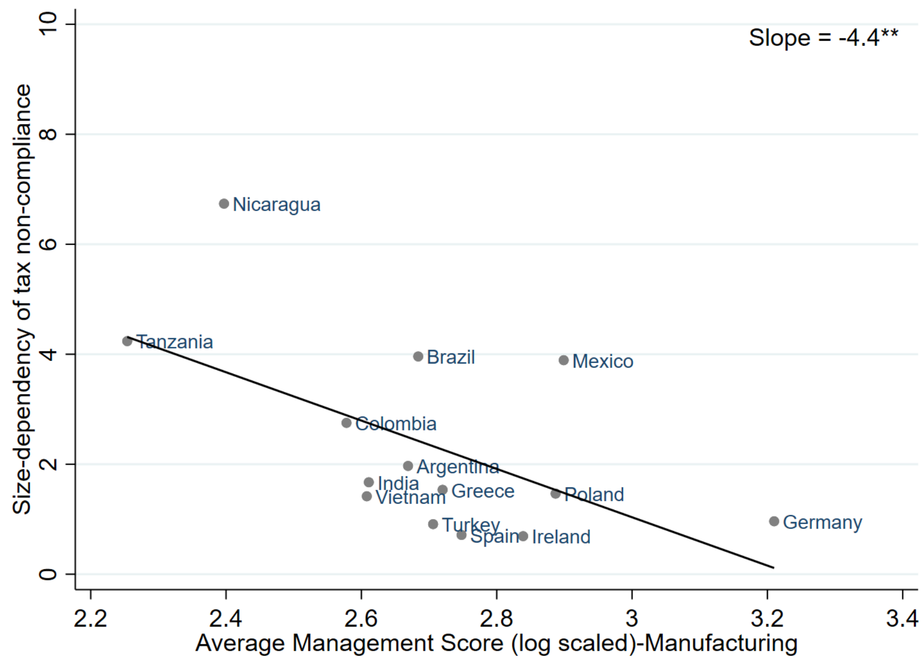
Figure 4: Size-Dependency and GDP per Worker



Sources: World Bank Enterprise Surveys, World Bank and author's calculations. **Notes:** This figure shows the relationship between the level of size-dependency tax noncompliance rate and the log GDP per worker at the cross-country level. Each dot represents a country. Level of size-dependency is defined as the absolute value of size coefficient(β_1) in equation 1. The solid line is the simple regression line where the size-dependency is the dependent variable and the GDP per worker is the independent variable. The slope of the regression line is -1.09 and it is statistically significant at the 1 percent level.

Managerial practices are better in countries with less size-dependent tax enforcement. Taking into account the observation that higher GDP per worker aligns with enhanced management practices and the accumulation of human capital among managers throughout their careers, a significant association also emerges between the degree of size-dependency in tax noncompliance and the quality of management. Figure 5 shows this relation. The management scores are taken from Bloom et al(2018) management practices dataset and all countries overlaping across this data set and WBES dataset are used.

Figure 5: Management Practices and Size-Dependency



Sources:World Bank Enterprise Surveys and author's calculations for tax noncompliance and Bloom et al. (2014) for average management scores of countries. **Notes:** This figure shows the relationship between the level of size-dependency of tax noncompliance rate and the average management score at the cross-country level. Each dot represents a country. Level of size-dependency is defined as the absolute value of size coefficient(β_1) in equation 1. The solid line is the simple regression line where the size-dependency is the dependent variable and management score is independent variable. The slope of the regression line is -4.4 and it is statistically significant at the 5 percent level.

3.3 Discussion

Robustness The preferred measure of wealth in this analysis is GDP per worker, as it aligns more consistently with the theoretical model to be presented in the next section. Figure A2 illustrates that the observations, such as establishments in wealthier countries displaying higher compliance with the tax code and that tax noncompliance size-dependency is lower in rich countries, remain consistent regardless of the choice of wealth measure. These relationships persist when alternative wealth measures, such as purchasing power parity-adjusted GDP per capita and real GDP per capita, are employed.

The countries in my final sample vary a lot in terms of their populations (from Cabo Verde with a total population of 460.000 to China with a total population of 1.3 billion in the year that samples are collected). One may be concerned that weighting very large and very small countries equally drives the results. To mitigate this concern, I execute the same analysis by assigning population weight to each observation. The results are summarized in Figure A3. The relation of GDP per worker with average tax-non compliance and level of size-dependency of tax noncompliance survive under population weighed sample.

To ensure representativeness, the WBES employs a stratified random sampling method, stratifying establishments by size, sector, and geographic region within each country. In the earlier survey years, no sampling weights were assigned to individual observations, relying solely on the stratified random sampling of the survey. For consistency across my final sample, I report results without sampling weights. To test the robustness of the relationship between the level of size-dependency and GDP per worker, I examined a subsample of countries that have available sampling weights. Figure A4 illustrates that the relationship between the level of size-dependency and GDP per worker becomes even stronger within this subsample.

The results suggest a correlation between size-dependency and GDP per worker. However, some might argue that a third factor, such as the quality of institutions, could be the primary driver of this correlation. To address this concern, I examined the relationship between these two variables while controlling for commonly used World Bank Governance Indicators, including Government Effectiveness, Regulatory Quality, Rule of Law, and Control of Corruption. Although the relationship is slightly weakened, the negative association between size-dependency and GDP per Worker remains statistically significant.[12]

Reverse Causality and Omitted Variable Bias Problems It's important to emphasize that the results reveal a conditional correlation between tax noncompliance and establishment size in each country in the sample. However, it's challenging to establish a cause-and-effect relationship solely from this analysis. Omitted variable bias and reverse causality pose significant threats to making causal statements based on equation 1. For instance, it could be the case that larger establishments choose to comply more with the tax code because they are subject to better monitoring. Alternatively, establishments might reduce their size intentionally to lower their tax liabilities.

To address these issues, Bachas et al. (2019) employed an innovative approach using the average size of firms in industries in the US as an instrument for the average size of firms in the same industry in lower-income countries, circumventing the challenges of aggregating tax noncompliance data at the establishment level. Their results suggest that an increase in the size-ranking of an industry leads to lower average tax noncompliance in that industry.

[12]The slope of the regression line decreases from -1.09 to 0.93 after adding institutional quality controls to the regression

Moreover, the magnitude of the size-tax compliance gradient is more pronounced for groups of wealthier income countries. Indeed, while Bachas et al. (2019)'s analysis operates at the industry level, the essence of their findings harmonizes closely with the stylized facts presented earlier in this study.

Reference Point Problem An important caveat to consider in this analysis is that the tax noncompliance measure relies on managers' reported beliefs about the tax noncompliance rate of "typical establishments in their area of business." This approach assumes that managers accurately report their own tax compliance based on the typical establishment's behavior. However, it's possible that when answering the survey question, a manager might use "a typical establishment" as a reference point rather than accurately reflecting their own establishment's situation. This could introduce bias into the results, particularly if the reporting manager's establishment significantly differs in size or other characteristics from the "typical establishments in her area of business.

The World Bank economists use this particular method of asking questions to encourage managers to provide truthful responses, especially when the questions are sensitive, such as those related to illegal activities like tax evasion, bribery, and other illicit practices within establishments. However, the effectiveness of this approach remains somewhat mysterious.

One recent study conducted by Aga and Jolevski (2023) sheds some light on this issue. They utilized data from one of the enterprise surveys conducted in Zambia to investigate whether asking sensitive questions using different phrasing, such as referring to "this establishments" or "establishments like this," would yield different responses. Interestingly, their findings showed that approximately 88 percent of managers provided the same answer, regardless of how the question was framed.

In addition to individual-level evidence, aggregate-level data also supports the idea that tax compliance behavior is size-dependent. For instance, the IRS reports that the audit coverage rate for tax returns with gross income less than $200,000 is approximately 1%. This rate gradually increases for higher-income groups, reaching 9.77% for returns with gross income exceeding $10,000,000.[13] Similarly, in the UK, the estimated tax gap is 3% for large businesses, 6% for mid-size businesses, and 13% for small businesses.[14] The Australian Taxation Office's data reveals that small businesses paid around 87% of the total theoretical tax payable in 2018–19, while this figure rises to approximately 96% for large businesses.[15]

If we consider the size-dependency of tax noncompliance as an incidence, the potential

[13]see U.S. Department of the Treasury, Internal Revenue Service (2009)

[14]see Her Majesty's Revenue and Customs (2019)

[15]See Australian Taxation Office (2020)

bias related to managers' reference points actually leans toward reporting tax noncompliance rates that align with the average behavior of the economy. In other words, managers of both smaller and larger establishments may report tax noncompliance rates that are closer to the typical or 'average' establishments. Therefore, the level of size-dependency discussed in this section should be viewed as a conservative estimate, suggesting that the actual size-dependency in tax noncompliance could be even higher."

4 Model

The model is built upon the life cycle version of Lucas (1978) span-of-control model, where managers can invest in their skills, following the approach in Guner et al. (2018). The key innovation is that managers decide how much of their sales to conceal from the government, factoring in the risk of being inspected by tax officials. This inherent inspection risk introduces stochastic elements into the model.

Initially, I will present this general stochastic version of the model. Then, by assuming a specific functional form for the inspection probability, I will simplify the model into a deterministic setup for analytical tractability. This deterministic version will be used for the quantitative analyses. Importantly, this setup allows for the coexistence of establishments with varying sizes and tax noncompliance rates in the stationary equilibrium, which are the primary concerns of this study.

4.1 General Model

In each period, a cohort of individuals is born with an initial endowment of capital k_0 and managerial ability z. The population growth rate is g, meaning that each new cohort is $1+g$ times the size of the preceding cohort. Each individual's lifespan is J periods. Initial managerial ability follows a probability density function $f(z)$ and cumulative distribution function $F(z)$. Each individual has one unit of time until retirement at age J_R. At the start of their life, individuals make a one-time choice between two occupations: worker or manager. This occupation choice is irrevocable, and those who choose a specific occupation supply their labor inelastically in that field until retirement.

Each individual's objective is to maximize the expected present value of lifetime utility from consumption.

$$E_0 \sum_{j=1}^{J} \beta^{j-1} u(c_j) \tag{2}$$

where $\beta \in (0,1)$ is discount factor and c_j represents the consumption of age-j individual. The utility function u is defined as $u(c) = log(c)$. Note that the expectation term in the preference comes from the uncertainty related to tax audits.

Government Government plays two crucial roles in the economy. First, it imposes a proportional output tax, denoted as $\tau \in (0,1)$, on each establishment to finance public spending (G). However, managers may not fully comply with this tax code; they may hide a portion $e \in (0,1)$ of their total output from the government. This hidden portion is referred to as the tax noncompliance rate. The second role of the government is to audit the manager's compliance with the tax code. However, audits are resource-intensive, and the government's resources are limited. To address this, tax administrators do not audit every establishment. Instead, they perform random inspections to monitor compliance. Each manager faces a probability of being audited, denoted as $p(z,e)$, where z represents managerial ability, and e is the fraction of output concealed from tax authorities. There are two key assumptions on the probability function: the probability of getting inspected is a non-decreasing function of managerial ability and tax noncompliance rate. If an establishment is inspected and tax noncompliance is detected, the manager has to pay taxes and a fine, which is proportional to the total tax concealed. M represents the fine rate.[16]

Managers A manager operates a firm by using her managerial ability. Given factor prices wage, w and the risk-free rate of return, r, she chooses how much capital, k and labor, n to hire every period. Managers have access to the following span of control technology:

$$f(z,k,n) = Az^{1-\gamma}(k^{\alpha}n^{1-\alpha})^{\gamma} \tag{3}$$

where $\gamma \in (0,1)$ is the span of control parameter, implying that the production technology exhibits decreasing returns to scale. A is the aggregate productivity term that is common to all establishments and it is normalized to one for the rest of the paper.

Also, each manager chooses a tax noncompliance rate $e \in (0,1)$. If her establishment is not subjected to inspection, the manager's profit is realized as follows:

$$ef(z,k,n)+(1-\tau)(1-e)f(z,k,n)-wn-(r+\delta)k = (1-\tau+\tau e))f(z,k,n)-wn-(r+\delta)k \tag{4}$$

where the $\delta \in (0,1)$ is the capital depreciation rate. The first term $ef(z,k,n)$ stands

[16] In other words, the fine is equal to $M \times e \times output$.

for tax-free fraction of output, and the second term, $(1-\tau)(1-e)f(z,k,n)$ represents the after-tax part of the reported output. Similarly, a manager earns the following profit in the case of getting inspected:

$$(1-\tau)f(z,k,n)-Mef(z,k,n)-wn-(r+\delta)k=(1-\tau-Me))f(z,k,n)-wn-(r+\delta)k \quad (5)$$

where the first term, $(1-\tau)f(z,k,n)$ is the after-tax output and second term, $Mef(z,k,n)$ is the penalty proportion to the total amount of non-reported output. If a manager chooses to fully comply the tax code, i.e. $e=0$, she gets $(1-\tau)f(z,k,n)-wn-(r+\delta)k$ no matter inspection is realized or not.[17]

After a manager observing whether her establishment is inspected or not, she decides how much of her income to allocate towards current consumption, c, savings, a', and investment in improving her managerial skills, x. The law of motion for managerial ability is assumed to be the following functional form:

$$z_{j+1}=(1-\delta_z)z_j+g(z_j,x_j)=(1-\delta_z)z_j+B_j z_j^{\theta_1}x_j^{\theta_2} \quad (6)$$

where z_{j+1} represents age-$j+1$ managerial ability and $\theta_1,\theta_2\in(0,1)$. δ_z stands for the depreciation of skills over time. B_j is the overall efficiency of investment in skills and I adopt the assumption, as outlined in Guner et al. (2018), that $B_j=(1-\delta_\theta)B_{j-1}$ with an initial value of $B_1=\theta$. Under this functional form and the restriction on θ_2, skill accumulation technology satisfies three important properties: (i) it exhibits complementary between current ability and investments in managerial ability, i.e., $g_{zx}>0$, (ii) investment in ability is an essential input for skill accumulation technology, i.e., $g(z,0)=0$, (iii) the technology shows diminishing returns to skill investments, i.e., $g_{xx}<0$.

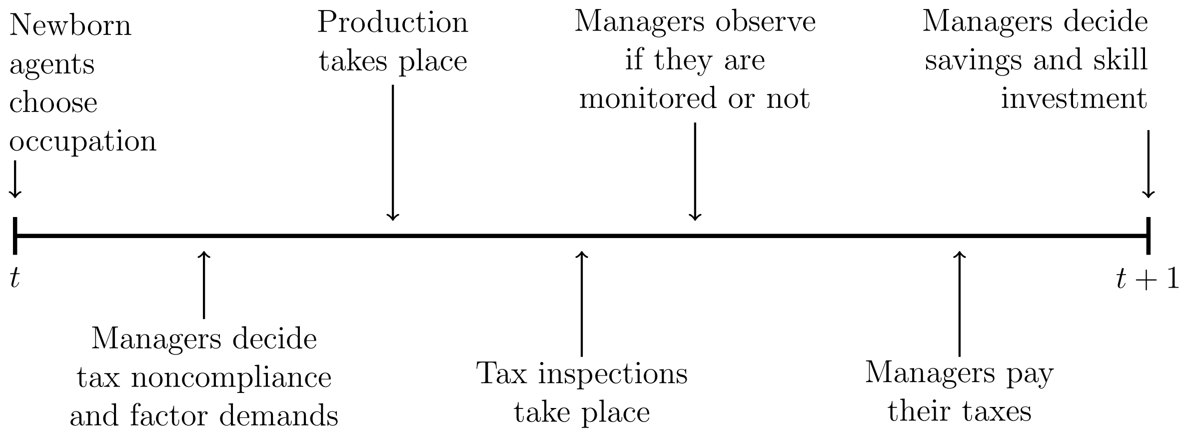
Workers In this model, there is no heterogeneity among workers. They supply their labor endowment inelastically and earn the market wage, regardless of their initial managerial ability. A worker's objective is to optimally allocate her income between current consumption and savings each period.

4.1.1 Decisions

Managers In this model, managers first decide their tax noncompliance rate and factor demands simultaneously. After observing whether their establishment is inspected, they

[17]Note that I use the terms "monitored," "audited," "inspected," or "get caught" interchangeably

Figure 6: Timing of Events



allocate their disposable income towards consumption, saving, and investment in managerial skills. Figure 6 summarizes the timing of events for the managers in the model. The dynamic programming problem of a manager of age-j is the following:

$$V_j(z,a) = \max_{e\in[0,1],k,n}\{p(z,e)V_j^d(z,a,e,k,n) + (1-p(z,e))V_j^n(z,a,e,k,n)\} \tag{7}$$

where $V_j^d(z,a,e,k,n)$ is the value function for the case that the establishment is inspected which is given by:

$$\begin{aligned} & V_j^d(z,a,e,k,n) = \max_{a',x\geq 0}\{log(c) + \beta E[V_{j+1}(z',a')]\} \\ s.t \qquad & c + a' + x = (1-\tau-Me)f(z,k,n) - wn - (r+\delta)k + (1+r)a \qquad \forall 1\leq j\leq J_R \\ & c + a' = (1+r)a \qquad \forall j > J_R \\ & z' = (1-\delta_z)z + g(z,x) \qquad \forall 1 \leq j \leq J_R \end{aligned} \tag{8}$$

and similarly, $V_j^n(z,a,e,k,n)$ is the value function for the the case that inspection is not realized which is given by:

$$\begin{aligned} & V_j^n(z,a,e,k,n) = \max_{a',x\geq 0}\{log(c) + \beta E[V_{j+1}(z',a')]\} \\ s.t \qquad & c + a' + x = (1-\tau+\tau e)f(z,k,n) - wn - (r+\delta)k + (1+r)a \qquad \forall 1\leq j\leq J_R \\ & c + a' = (1+r)a \qquad \forall j > J_R \\ & z' = (1-\delta_z)z + g(z,x) \qquad \forall 1 \leq j \leq J_R \end{aligned} \tag{9}$$

with

$$V_{J+1} = \begin{cases} 0, & if \quad a \geq 0 \\ -\infty, & otherwise \end{cases} \tag{10}$$

Workers The dynamic programming problem of a worker of age-j is the following:

$$\begin{aligned} W_j(a) &= \max_{a'} \{log(c) + \beta W_{j+1}(a')\} \\ c + a' &= w + (1+r)a \qquad \forall 1 \leq j \leq J_R \\ c + a' &= (1+r)a \qquad \forall j > J_R \end{aligned} \tag{11}$$

with

$$W_{J+1} = \begin{cases} 0, & if \quad a \geq 0 \\ -\infty, & otherwise \end{cases} \tag{12}$$

4.2 Deterministic Case

In the stochastic model environment, managers make simultaneous decisions regarding their tax noncompliance rate, denoted as e, and their factor demands, namely capital k and labor n (as shown in Equation 7). Given that managers are willing to take varying levels of risk at different ages, their choice of tax noncompliance rate becomes age-dependent. Consequently, their demands for capital and labor also vary with age. In this section, I introduce a specific functional form for the probability of getting inspected, which implies that no establishment will be detected in the equilibrium. However, managers can still determine their tax noncompliance rate. Consequently, the factor demands become independent of the age of managers, making the model a more analytically tractable framework for studying tax noncompliance phenomena. It's crucial to emphasize that the deterministic model is simply a specialized case of the stochastic model. I will employ this version of the model for quantitative analysis.

Probability of getting inspected Specifically, I assign the following functional form for the probability function:

$$p(z,e) = \begin{cases} 0 & if \quad e \leq h(z) \\ 1 & if \quad e > h(z) \end{cases} \tag{13}$$

where $h(z) = \dfrac{1}{z\eta + \rho}$ with $\rho > 1$ and $\eta > 0$. In this monitoring technology, the government sets a threshold level of tax noncompliance rate for all establishments based on their size (or the ability of the managers operating them). If an establishment opts to conceal more output than the assigned threshold level, the government conducts monitoring and imposes penalties on that establishment. Conversely, if the establishment's tax noncompliance rate falls below the threshold, the government overlooks the tax noncompliance.

The threshold levels are determined by the function $h(z)$, which decreases as establishment size increases. This choice of the probability of being monitored is based on the concept that larger establishments are more conspicuous to tax authorities, meaning that even a relatively minor degree of tax noncompliance by these larger establishments is more likely to come under the scrutiny of tax authorities.

The primary advantage of this functional form is that the parameters, η and ρ, pin down the optimal tax noncompliance rate for all establishments in the economy. Specifically, for a given managerial ability z, choosing a tax noncompliance rate less than $(z\eta + \rho)^{-1}$ is suboptimal. This is because selecting a slightly higher noncompliance rate does not increase the probability of being inspected while it reduces the effective tax rate. Consequently, $h(z)$ establishes a lower bound for the optimal tax noncompliance rate, i.e., $e(z) \geq h(z)$. Conversely, managers do not choose a noncompliance rate higher than $h(z)$, ensuring that they will be inspected and penalized by tax authorities. This enforcement mechanism compels managers to opt for low tax noncompliance rates to avoid monitoring by tax officials. Therefore, $h(z)$ serves as an upper bound for the optimal tax noncompliance, i.e., $e(z) \geq h(z)$. In summary, the optimal noncompliance rate is exclusively determined by the parameters of $h(z)$:

$$e^*(z) = \frac{1}{z\eta + \rho} \tag{14}$$

Because the choice of the tax noncompliance rate is directly determined by managerial ability, the factor demands become dependent solely on managerial ability and factor prices. Consequently, the stochastic model explained above simplifies to a more analytically tractable version.

It is important to highlight some critical properties of this probability function. First, ρ serves as the *intercept parameter* that determines the tax noncompliance rate for establishments managed by individuals with zero managerial ability, i.e., $e^*(0) = \frac{1}{\rho}$. A higher value of ρ implies a lower noncompliance rate for all firms. In essence, as ρ increases, the government's capacity to enforce taxes becomes stronger. If we consider a scenario with an arbitrarily large intercept parameter ρ, tax noncompliance for all levels of managerial ability tends to zero, creating an environment of full tax enforcement.

The *slope parameter* η dictates the level of size-dependency in tax noncompliance. When

$\eta = 0$, this model implies that all managers, regardless of their skill levels, exhibit the same tax noncompliance rate. For example, when $\eta = 0$ and $\rho = 2$, the tax noncompliance rate is 50% for all establishments in the economy. However, when $\eta > 0$ for a given intercept parameter, the partial derivative of e^* with respect to z is negative, indicating that tax noncompliance is size-dependent.[18]

4.2.1 Decisions

As previously emphasized, in the deterministic setup, factor demands and after-tax income of managers are age-independent and depend solely on managerial ability and factor prices. Therefore, the after-tax income of a manager with ability z is given by:

$$\Pi(z) = \max_{k,n} \Big\{(1-\tau+\tau e^*(z))z^{1-\gamma}(k^{\alpha}n^{1-\alpha})^{\gamma} - wn - (r+\delta)k\Big\} \tag{15}$$

where $e^*(z) = \frac{1}{z\eta+\rho}$ is the optimal decision for tax noncompliance rate of a manager with ability level z. The solution of this static problem gives the factor demands as the followings:

$$k(z) = [\gamma(1-\tau+\tau e^*(z))]^{\frac{1}{1-\gamma}} \left(\frac{a}{r+\delta}\right)^{\frac{(a-1)\gamma+1}{1-\gamma}} \left(\frac{w}{1-\alpha}\right)^{\frac{(a-1)\gamma}{1-\gamma}} z \tag{16}$$

$$n(z) = [\gamma(1-\tau+\tau e^*(z))]^{\frac{1}{1-\gamma}} \left(\frac{a}{r+\delta}\right)^{\frac{a\gamma}{1-\gamma}} \left(\frac{w}{1-\alpha}\right)^{\frac{a\gamma-1}{1-\gamma}} z \tag{17}$$

Substituting factor demands into the profit function gives after-tax profit(or income) of a manager with ability z:

$$\Pi(z) = \Delta[1-\tau+\tau e^*(z)]^{\frac{1}{1-\gamma}} z \tag{18}$$

where Δ is a combination of prices and parameters.[19]

The functional form of probability of getting inspected reduces the managers problem to the following:

$$\begin{aligned}
& V_j(z,a) = \max_{a',x\geq 0}\{log(c) + \beta V_{j+1}(z',a')\} \\
s.t \quad & c + a' + x = \Pi(z) + (1+r)a \qquad \forall 1 \leq j \leq J_R \\
& c + a' = (1+r)a \qquad \forall j > J_R \\
& z' = (1-\delta_z)z + g(z,x) \qquad \forall 1 \leq j \leq J_R
\end{aligned} \tag{19}$$

[18] $\frac{\partial e^*}{\partial z} = -\frac{\eta}{(\eta z+\rho)} < 0$

[19] $\Delta = \left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha\gamma}{1-\gamma}} \left(\frac{1-\alpha}{w}\right)^{\frac{\gamma(1-\alpha)}{1-\gamma}} (1-\gamma)^{\frac{1}{1-\gamma}}$

The interior solution to the managers' problem can be characterized by two conditions. The first condition is the standard Euler equation that characterizes the inter-temporal decision between today's consumption and the next period's asset:

$$\frac{1}{c_j} = \beta(1+r)\frac{1}{c_{j+1}} \qquad \forall 1 \leq j \leq J-1 \tag{20}$$

The second condition is a no-arbitrage condition for asset and investment in managerial ability.

$$(1+r) = g_x(z_j, x_j)\Pi_z(z_{j+1}) + \frac{g_x(z_j, x_j)}{g_x(z_{j+1}, x_{j+1})}[g_z(z_{j+1}, x_{j+1}) + 1 - \delta_z] \qquad \forall 1 \leq j \leq J_R - 1 \tag{21}$$

The left-hand side of this equation represents the opportunity cost of investing in managerial skills. A manager could earn $(1+r)$ units of the consumption good by investing one unit in physical assets rather than investing one unit in managerial skills. The right-hand side of the equation is the net marginal benefit of skill investment. The first term represents the benefit deduced from the next period profit generated by additional managerial ability. An extra unit of skill investment in the current period is transformed into next period managerial skill with $g_x(z, x_j)$, and $\Pi_z(z, r, w)$ transforms the additional managerial ability into profit. Also, an extra unit of investment in managerial ability in the current period relaxes the skill accumulation technology constraint in the subsequent period. The second term on the right-hand side represents this relaxation. This term disappears in the last working age period of managers, and the equation simplifies to the following for $j = J_R - 1$.

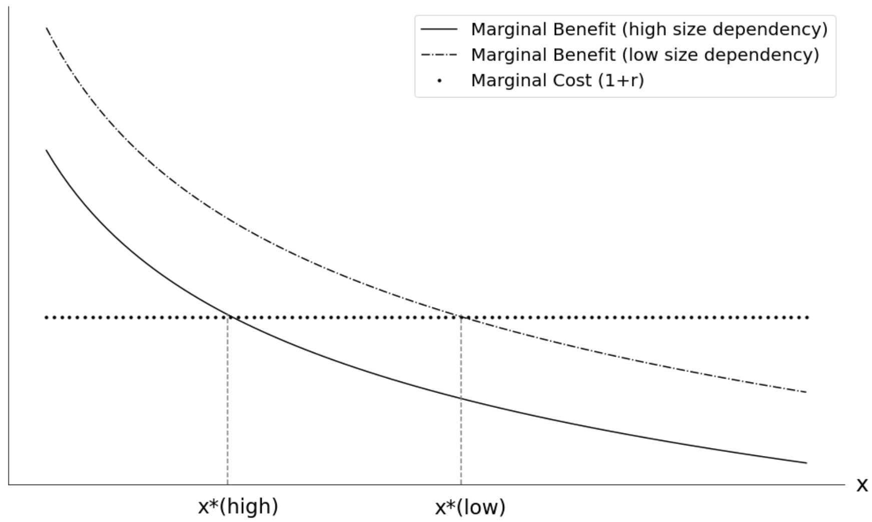
$$(1+r) = g_x(z_j, x_j)\Pi_z(z_{j+1}) \tag{22}$$

Tax noncompliance and skill investment decision The distortionary effect of size-dependent tax noncompliance is embedded in $\Pi_z(z)$ in equation 21, which is the partial derivative of after-tax profit with respect to managerial ability. Let's examine this term more closely. It can be explicitly written as follows:

$$\Pi_z(z) = \Delta[1 - \tau + \tau e^*(z)]^{\frac{1}{1-\gamma}} + \Delta[1 - \tau + \tau e^*(z)]^{\frac{\gamma}{1-\gamma}} e_z^*(z) z \tag{23}$$

Higher managerial ability implies higher output. The first term on the right-hand side represents the benefit of this additional output. Simultaneously, higher managerial ability implies a lower level of tax noncompliance rate as well as a higher level of effective tax. The

Figure 7: Determination of Skill Investment



effective tax cost of an additional unit of managerial ability is represented by the second term on the right-hand side. Note that this term has a negative sign since $e_z^*(z) < 0$ as long as η is positive. If there is no size-dependency in tax noncompliance, i.e., $e_z^*(z) = 0$, the second term disappears, and an additional unit of managerial ability yields more benefit.

For simplicity, let's consider a two-period version of the model. In this case, the second term on the right-hand side of equation 21 disappears since there is no subsequent period. Figure 7 illustrates the optimal decision for skill investment in this simplified model for low and high levels of size-dependency. An increase in size-dependency shifts the marginal benefit curve to the left due to the additional cost derived from lower tax noncompliance, leading to a reduction in skill investment. In brief, size-dependency in noncompliance creates a disincentive for managers to invest in their skills. They anticipate being audited more thoroughly, resulting in a higher effective tax rate when operating a larger establishment with higher managerial ability.

Discussion I assume that the probability of being audited depends on managerial ability, denoted as z, and the share of concealed income, represented as e. Both of these factors are arguably not observable to tax authorities. The assumption that the probability increases as e rises, which was first introduced by Allingham and Sandmo (1972), is a reduced form notion of the idea that the more taxes are evaded, the greater the likelihood of detection becomes. While tax evasion inherently involves secret actions that remain hidden from tax authorities, most tax agencies employ some form of risk management system. These systems are designed to allocate resources effectively, prioritizing cases where enforcement actions are likely to yield the best results Slemrod (2019) .

The probability depending on managerial ability, which is a productivity measure that may not be observed by the government, has been modeled differently in other papers. For example, Bigio and Zilberman (2011) use employment as an input for the probability function while Leal Ordóñez (2014), and Di Nola et al. (2021) use capital demand as an input. However, assuming that these factors are observable to tax authorities would be optimistic, especially in the context of developing countries. Bigio and Zilberman (2011) focus on the U.S., where formal employment is prevalent, making it a suitable choice. However, in many developing countries, informal employment constitutes a significant portion of the labor force, making social security payrolls less meaningful to tax authorities. Leal Ordóñez (2014) uses capital based on the observation that capital, in particular, is harder to hide. Nevertheless, even if capital is easier for the government to observe, it can be very challenging to measure the capital stock (Syverson (2011)).

While we may not have precise information about which size measure tax authorities are targeting, there is evidence suggesting that they are auditing larger establishments more frequently, and larger establishments tend to exhibit higher compliance with the tax code. The advantage of incorporating managerial ability an input for the probability function is that there is a one-to-one mapping from managerial ability, denoted as z, to various potential firm size measures, such as the number of workers (n), capital amount (k), output (y), and profit (Π). Additionally, if the probability of detection depended directly on labor or capital input choices, it would distort firms' decisions in favor of the other factor which is not primary interest of this paper.

4.3 Equilibrium

In the standard Lucas span-of-control model, the value of being a manager is an increasing function of initial managerial ability in the sense that higher managerial ability brings higher profit from operating a firm due to the decreasing returns-to-scale in production technology. On the other hand, the value of being a worker does not depend on managerial ability. These properties of value functions guarantee the existence of a threshold, z^* at which a newborn individual is indifferent between being a manager and a worker.[20] In this model where tax noncompliance is a decreasing function of managerial ability, the existence of such

[20]Assuming that all individuals are born with zero assets, this threshold is determined by: $V_1(z, 0) = W_1(0)$.

a threshold is not guaranteed. But, the functional form of probability of getting inspected specified above together with the assumption on *intercept parameter* ($\rho > 1$) are sufficient for having a threshold deciding the occupations. The detailed derivation can be found in Appendix.

Now, consider the size of population is normalized to one. As I mentioned earlier, the first period managerial ability is distributed according to pdf $f(z)$ and cdf $F(z)$. Let managerial abilities and asset values take values in sets $\mathcal{Z} \in [0, \overline{z}]$ and $\mathcal{A} \in [-\overline{a}, \overline{a}]$, respectively. Let $\psi_j(\tilde{a}, z)$ be the mass of age-j agents with asset level $\tilde{a}$ and managerial ability z. Let

$$\tilde{f}_j(z) = \int_{-\overline{a}}^{\overline{a}} \psi_j(\tilde{a}, z) d\tilde{a} \tag{24}$$

be skill distribution for age-j agents. Consider the following policy functions: the next period asset holding of age-j workers, $g_j^W(a)$, the next period asset holding of age-j managers, $g_j^M(a, z)$, the skill investment decision of age-j managers, $x_j(a, z)$, and factor demands, $k(z)$, $n(z)$.

In the equilibrium, workers and managers solve their problems, and all markets clear. Specifically, a competitive equilibrium consists of value functions $\{V_j(z, a), W_j(a)\}_{j=1}^{J} + 1$, policy functions $\{g_j^W(a), g_j^M(a, z), x_j(a, z)\}_{j=1}^{J}$, $k(z)$, $n(z)$, input prices r, w and a tax non-compliance scheme $e^*(z)$ such that:

1. Given prices r, w, the value functions $\{W_j(a)\}_{j=1}^{J}$ and policy functions $\{g_j^W(a)\}_{j=1}^{J}$ solve worker's problem described in equation (11).

2. Given prices r, w and tax noncompliance scheme $e^*(z)$, the value functions $\{V_j(a, z)\}_{j=1}^{J}$ and policy functions $\{g_j^M(a, z), x_j(a, z)\}_{j=1}^{J}$, $k(z)$, $n(z)$ solve manager's problem described in equation (19).

3. The capital and labor markets clear. The market clearing condition for labor market is

$$\sum_{j=1}^{J_R-1} \int_{z^*}^{\bar{z}} n(z) \tilde{f}_j(z) dz = \sum_{j=1}^{J_R-1} F(z^*) \tag{25}$$

where the left hand-side stands for labor demand while the right hand side is the labor supply. The capital market clearing condition is

$$\sum_{j=1}^{J_R-1} \int_{z^*}^{\bar{z}} k(z)\tilde{f}_j(z)dz = \sum_{j=1}^{J-1} \left[\int_0^{z^*} \int_{-\bar{a}}^{\bar{a}} g_j^W(a)d\psi_j(a,z) + \int_{z^*}^{\bar{z}} \int_{-\bar{a}}^{\bar{a}} g_j^M(a,z)d\psi_j(a,z) \right] \tag{26}$$

where the left hand-side is capital demand and the right hand-side stands for the net capital supply of workers and managers. Also, goods market clearing condition holds by Walras' Law.

It is worthwhile to note that, as is discussed in Guner et al. (2018), this model framework offers a notion of *average managerial quality*, which is the summation of managerial abilities of all managers in the economy divided by the number of managers.

5 Calibration

A full tax enforcement is achieved in the model when the *intercept parameter* ρ is arbitrarily large. In this scenario, the probability function forces the tax noncompliance rate to be zero for all establishments in the economy, regardless of the manager's ability. This serves as the benchmark for the model. I calibrate the benchmark model parameters in order to match the U.S. plant size and U.S managerial income growth moments by assuming that the U.S government can fully enforce the tax code. I closely follow the calibration procedure used in the paper by Bhattacharya et al. (2013), as my benchmark without tax noncompliance is very similar to their model.

The model period corresponds to 5 years, and each individual enters the model at age 25, exiting at age 79. Thus, all individuals live for 11 model periods. The retirement age is set at 65, meaning individuals spend eight periods of their life in the labor force as either a manager or a worker, and the final three periods out of the labor force as retired. Initial managerial abilities follow a log-normal distribution characterized by a mean μ and a standard deviation σ.

In the model, there are 13 parameters in total: discount factor (β), depreciation rate of capital (δ), importance of capital (α), returns to scale (γ), mean and standard deviation of log managerial ability (μ and σ), population growth rate (g), output tax rate (τ) and five skill accumulation technology parameters (δ_z, θ, θ_1 and θ_2, δ_θ).

Parameters set outside the model According to Guner et al. (2008), the capital-output ratio, the capital share, and the investment-output ratio for the U.S. economy between 1960-2000 were reported as 2.325 (at the annual level), 0.326, and 0.178, respectively. These values

Table 2: Parameter Values (Annualized)

Parameter	Value
Depreciation rate (δ)	0.077
Population growth rate (g)	0.011
Importance of capital (α)	0.421
Output tax rate (τ)	0.200
Mean log-managerial ability (μ)	0
Skill accumulation technology (δ_z)	0.048
Returns to Scale (γ)	0.748
Dispersion in Log-managerial Ability (σ)	2.740
Discount Factor (β)	0.941
Skill accumulation technology (θ_1)	0.701
Skill accumulation technology (θ_2)	0.512
Skill accumulation technology (θ)	0.904
Skill accumulation technology (δ_θ)	0.059

Notes: The table displays model parameters together with their values. The parameters set outside the model are on the top panel, while the jointly calibrated parameters are listed in the bottom panel. All values are presented at the annual level.

are instrumental in determining the depreciation rate (δ) and the importance of capital (α). The steady-state law of motion for capital leads to a calculated annual depreciation rate (δ) of 0.077, which corresponds to 0.33 for the model period (5 years).[21] The importance of capital (α) is determined by considering the returns to scale parameter (γ) and the capital share. Given that the capital share in the model is represented as $\alpha\gamma$, we set α to 0.421 based on these considerations.[22] Furthermore, the population growth rate parameter (g) is set to match the long-run U.S. population growth rate, which is approximately 1% and is based on data from 1961 to 2021 sourced from the World Bank. Finally, the output tax rate (τ) is fixed at 0.2, in accordance with data from Monthly Treasury statements indicating that the federal government-to-GDP ratio in fiscal year 2022 stands at 20%. I borrow the depreciation rate of skills (δ_z) from Guner et al. (2018). Their theory assumes that managers stop investing in their skills toward the end of their working lives, which leads to an estimated δ_z of 0.048 at the annual level, equivalent to 0.217 for the model's 5-year period. Additionally, I normalized the mean of log managerial ability (μ) to 0.

[21] The steady-state law of motion for capital equates the depreciation rate to the investment-output ratio divided by the capital-output ratio. The depreciation rate for the model period (5 years) is obtained as $(1-\delta) = (1-\delta_{annual})^5$.

[22] With the given capital share and the calibrated value of γ, we derive the importance of capital (α) as 0.421.

Table 3: Empirical Targets: Model and Data

Statistic	Data	Model
Capital Output Ratio	2.32	2.31
Mean Size	15.8	15.7
Fraction of Small Establishments	0.86	0.87
Fraction of Large Establishments	0.03	0.03
Employment Share of Large Establishments	0.46	0.45
Relative Earning Growth (40-44/25-29)	0.17	0.17
Relative Earning Growth (60-64/25-29)	0.22	0.22

Notes: The table displays targeted moments together with their model counterparts in the benchmark calibration. *Small establishments* refer to establishments with fewer than 20 workers, while *large establishments* have 100 or more workers. *Relative Earning Growth (40-44/25-29)* stands for the growth rate of managerial incomes relative to those of non-managers between ages 25-29 and 40-44.

Parameters set in the model equilibrium I have seven remaining parameters to determine: β, γ, σ, θ, θ_1, θ_2 and δ_θ. I jointly calibrated these parameters to match the following seven moments of the U.S. economy: (i) aggregate capital-output ratio, (ii) mean establishment size, (iii) fraction of establishments with less than 20 workers, (iv) fraction of establishments with more than 100 workers, (v) fraction of the labor force employed in establishments with 100 or more workers, (vi) the growth rate of managerial incomes relative to those of non-managers between ages 25-29 and 40-44, (vii) the growth rate of managerial incomes relative to those of non-managers between ages 25-29 to 60-64. I use data from the U.S. Census County Business Patterns between 2012-2017 for the plant size distribution and growth rate estimates of managerial incomes from Guner et al. (2018). The parameters and their calibrated values are presented in Table 2, while Table 3 provides a comparison between the targeted moments and their corresponding model outcomes.

Discussion As highlighted in Bhattacharya et al. (2013), U.S. plant size data exhibit two crucial characteristics. Firstly, the size distribution is notably right-skewed, with nearly 86% of plants employing fewer than twenty full-time workers. Secondly, despite only 2.6% of plants employing more than 100 workers, they contribute to 46% of total employment. The top panels of Figure 8 illustrate how the benchmark model successfully replicates these aspects of the U.S. plant-size distribution, along with various other moments not explicitly targeted. The model parameters governing the returns to scale (γ) and the standard deviation of the initial skill distribution (σ) are pivotal in determining the U.S. plant-size distribution in the model.

This paper primarily focuses on examining the interplay between managerial quality

Figure 8: Size Distribution, Employment Shares, and Managerial Income Growth: Untargeted Moments

Notes: The top panels of the figure depict the model's ability to replicate the U.S. plant size distribution and employment shares. Among these, the only moments explicitly targeted are the proportions of establishments with less than 20 workers, establishments with more than 100 workers, and the employment share of establishments with over 100 workers. The bottom panels showcase the model's performance in reproducing earnings growth among managers relative to non-managers across the life-cycle. In this case, the moments specifically targeted encompass the earnings growth of managers compared to non-managers between ages 25-29 and 40-44, as well as between ages 25-29 and 60-64.

and tax noncompliance. Within the model, investment in skill development stands as a critical factor influencing managerial quality. In a hypothetical situation where managers are restricted from investing in their skills, the average managerial quality would plummet by roughly 45% compared to the benchmark level. This decline in managerial quality equates to a 16% reduction in aggregate output. [23] Therefore, considering the paper's focus on managerial quality, it is imperative to concentrate on moments related to managerial income growth. The parameters associated with skill accumulation technology (θ, θ_1, θ_2, and δ_θ) play a pivotal role in shaping managerial income growth within the model. For instance, even a modest 10% reduction in θ_2 results in a substantial decline in earnings growth among managers compared to non-managers, decreasing from the benchmark level of 17% to 12%. This highlights the significance of these parameters in elucidating the dynamics of managerial income. Additionally, the lower panel of Figure 8 illustrates that the model effectively replicates untargeted moments of managers' age-earning profiles.

[23] This evaluation relies on a straightforward counterfactual analysis, which entails contrasting the outcomes of the benchmark economy with those of an economy where investments in skill development are forbidden. This analysis is conducted within a partial equilibrium framework.

6 Findings

In this section, I will present the main quantitative findings of the paper. Firstly, I introduce incomplete tax enforcement to the benchmark economy and quantify the importance of size-dependency in tax enforcement. The key observation from this analysis is that as the degree of size-dependency in tax noncompliance increases, there is a substantial decrease in output, average managerial quality, mean establishment size, and the growth of managers' income over the lifecycle.

Subsequently, I compare the benchmark economy with various economies that incorporate incomplete tax enforcement. In these economies, tax noncompliance-related parameters are calibrated to replicate the moments of tax noncompliance observed in each respective country. Specifically, I have selected Spain, Turkey, Brazil, and Angola to represent different combinations of the level and size-dependency of incompleteness. The effects become substantial when we apply the model to the data. For instance, transitioning from the U.S. calibrated benchmark to Brazil results in a significant reduction of approximately 23% in average managerial quality and a 3% decrease in aggregate output.

6.1 Incomplete Tax Enforcement

I study the effect of incomplete tax enforcement and the size-dependency of tax enforcement via the following function: $e(z) = \frac{1}{z\eta+\rho}$, which has been discussed in detail in model section. The benchmark economy with full tax enforcement can be thought of where intercept parameter ρ is arbitrarily large so that $e(z) \to 0$ for all levels of managerial abilities. The slope parameter η governs the degree of size-dependency of tax noncompliance. When $\eta = 0$, this formulation implies the same tax noncompliance rate for all establishments, regardless of their size. For instance, $\eta = 0$ and $\rho = 2$ results in tax noncompliance rate to be 50% for all establishments in the economy. In other words, under these set of parameters, all the establishments hide half of their output from the government, which is equivalent to reducing the output tax rate by half.

Table 4 shows the steady-state consequences for an array of values of the slope parameter η, under $\rho = 1.33$. I consider four levels of size-dependency; $\eta = \{0, 10^{-5}, 10^{-4}, 10^{-3}\}$. For each value of η, the table also presents the average effective tax rate ratio of small establishments (those with fewer than 10 workers) to large establishments (those with more than 100 workers). [24] Specifically, when $\eta = 10^{-5}$, small establishments face an average effective tax rate 1.3 times higher than that of large establishments, and this ratio increases

[24] Effective tax rate is the legal tax rate net of tax noncompliance.

Table 4: The Effect of size-Dependency in Tax Noncompliance

	Complete Tax Enforcement	Incomplete Tax Enforement			
	(1)	(2)	(3)	(4)	(5)
Slope Parameter (η)	0	0	10^{-5}	10^{-4}	10^{-3}
Effective Tax Ratio (>100 / <10)	1.0	1.0	1.3	2.3	2.8
Output	100.0	111.8	110.1	106.4	100.4
Share of Managers (%)	6.0	5.6	6.0	6.7	7.6
Average Managerial Quality	100.0	115.7	105.6	89.1	76.9
Investment in Skills (% Output)	0.51	0.80	0.65	0.45	0.38
Earning Growth (40-44/25-29)	16.5	25.2	20.4	11.9	8.5
Mean Size	15.7	16.7	15.7	14.0	12.1
Fraction of Large Estb.	2.6	2.7	2.6	2.3	1.6
Emp. Share of Large Estb.	45.0	48.5	44.8	35.6	29.3
Tax Revenue to Output Ratio	20.0	5.0	5.8	8.2	12.7
Revenue Neutrality					
Output	100.0	100.0	98.6	95.8	96.6
Average Managerial Quality	100.0	100.0	84.2	65.4	57.8
Mean Size	15.7	15.7	14.0	11.4	9.7
Earning Growth (40-44/25-29)	16.5	16.5	7.5	0.7	-0.2
Tax Revenue to Output Ratio	20.0	20.0	20.0	20.0	20.0

Notes: Entries show the effect on displayed variables associated with different levels of size-dependency in tax noncompliance (η). Corresponding η values are presented on the first entry of each column. Note $\eta = 0$ in column (2) implies that this economy is free of size-dependency. *Effective Tax Ratio (>100 / <10)* stands for the ratio of the average effective tax rate of the small establishments (less than 10 workers) to large establishments (more than 100 workers). *Earning Growth (40-44/25-29)* stands for the earning growth of managers between ages 25-29 and 40-44 relative to earning growth of workers between the same ages. *Emp. Share of Large Estb.* stands for the employment share of establishments that employ more than 100 workers. Output and average managerial quality at the benchmark economy are normalized to 100.

with higher values of η

To facilitate comparison, column (1) lists the outcomes of the benchmark economy with perfectly enforceable taxes. In column (2), I present the results of an economy with incomplete tax enforcement without size-dependency, and then introduce varying levels of size-dependency in columns 3 - 5.

Incomplete tax enforcement without size-dependency As expected, introducing incomplete tax enforcement without size-dependency leads to an increase in output and average managerial quality across steady states. This is because it is an equivalent case to reducing the distortionary output tax by one-fourth without introducing any additional sources of distortion. In quantitative terms, transitioning from a tax noncompliance-free benchmark economy to incomplete tax enforcement without size-dependency results in an approximately 12% increase in output, a 16% increase in average managerial quality, and an increase in mean size from 15.7 to 16.7.

The primary driver of this outcome is the reduction in the effective tax rate resulting from the opportunity for tax noncompliance. Managers invest more in their managerial skills due to the extra income resulting from the lower effective tax rate. The investment in managerial skill formation as a share of output increases from its benchmark value of 0.51% to 0.8% in the incomplete tax enforcement economy.

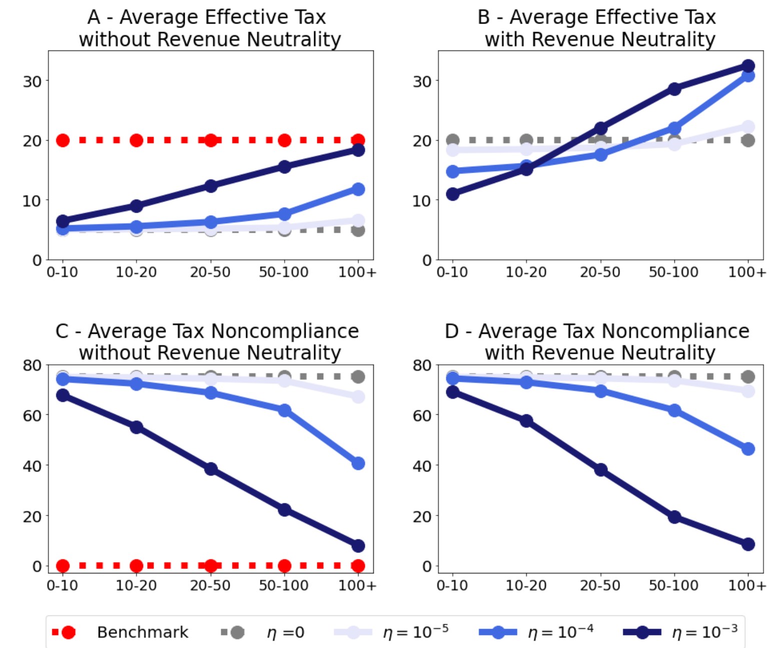
Incomplete tax enforcement with size-dependency Once the size-dependency is introduced to the model, the gains from size-dependency free incomplete tax enforcement diminish. Columns 3-5 in Table 4 show the steady-state results of the incomplete tax enforcement economies with various levels of size-dependencies. The critical observation is that as the degree of size-dependency increases, average managerial quality and output decreases gradually.

In an economy with high size-dependency (i.e., $\eta = 10^{-3}$), the average managerial quality is approximately 23% lower compared to the benchmark economy. This significant drop in managerial quality can be attributed to changes in both the extensive and intensive margins of managers. On the extensive margin, size-dependent tax enforcement leads to the selection of less capable individuals as managers. While the proportion of managers is 6% in the benchmark scenario, it increases up to 7.6% as size-dependency becomes more pronounced. On the intensive margin, managers are discouraged from investing in their skills due to size-dependent enforcement. In the benchmark model, managers invest as much as 0.51% of the output, but this decreases up to 0.38% under more size-dependent enforcement regimes. One natural consequence of this reduced investment in skills over the lifecycle is a flatter age-earning profile for managers. The benchmark value of a 16% earnings growth rate for managers from ages 20-25 to 40-45 is halved when $\eta = 10^{-3}$.

These variations in the quality of managerial talent have notable effects on establishment size and employment distributions within the economy. As the number of large establishments decreases, they also employ a smaller fraction of the workforce. Consequently, the mean establishment size in the economy decreases as size-dependency increases.

Revenue output ratio neutrality The decline in managerial quality is not accompanied by a drop in aggregate output. Even in a highly size-dependent economy, where the average effective tax rate of large establishments is 2.8 times that of small establishments,

Figure 9: Tax Noncompliance and Effective Tax Rate by Establishment Size



Notes: The Panel C and D display the average tax noncompliance rate of establishments without and with revenue neutrality, across different values of η as listed in Table 4. The x-axis represents the different establishment size categories. Panel A and B illustrate corresponding average effective tax rate, which is the legal tax rate net of for tax noncompliance, for establishments without and with revenue neutrality

the aggregate output level remains nearly the same as the benchmark level, despite a significant 23.1% decrease in average managerial quality. This counterintuitive result can be attributed to variations in effective tax rates across economies. In incomplete tax enforcement economies, the tax revenue-output ratio is well below the benchmark level of 20%. As demonstrated in Panel A of Figure 9, all establishments in these economies face lower effective tax rates than in the benchmark case.

To address the challenges of comparing incomplete tax enforcement economies with the benchmark due to varying tax revenue-output ratios, I extend this analysis to achieve tax revenue-output ratio neutrality. This is done by adjusting the flat output tax rate (τ) for each economy to fix the tax revenue-output ratio at its benchmark value of 20%. In revenue-neutral cases, the effective tax rate profiles shift upward while maintaining the same size-dependency, as shown in Panel B. Comparing tax noncompliance rates with and without neutrality (Panel C and D) reveals that neutralizing tax levels does not alter the size-dependency of tax noncompliance.

The bottom panel of Table 4 presents the findings of revenue-neutral economies. It's worth noting that in the context of revenue-output ratio neutrality, the incomplete tax enforcement economy without size-dependency is equivalent to the benchmark economy. When tax-output ratios are neutralized across economies, aggregate output responds to changes in size-dependency along with managerial quality. Transitioning from the benchmark economy to an economy with moderate size-dependency, specifically the one with $\eta = 10^{-4}$, results in a 4.2% reduction in output, a 35% decrease in average managerial quality, a drop in mean establishment size from 15.7 to 11.4, and nearly eliminates all growth in managerial income from ages 20-25 to 40-45.

6.2 From Model to Data

The quantitative analysis above demonstrates that the size-dependency of tax enforcement has a negative impact on managerial quality and aggregate output. To connect these findings with real-world data, I examine examples that compare the benchmark economy with various countries' economies. I've selected four countries, each representing different tax enforcement regimes: one high-income country, Spain; two upper-middle-income countries, Turkey and Brazil; and one lower-middle-income country, Angola.

In Spain, the tax noncompliance rate is exceptionally low, averaging only 3.7 percent. Additionally, tax noncompliance exhibits a low level of size-dependency with a coefficient of -0.7, indicating that in Spain, one log point increase in size corresponds to a 0.7 percentage point decrease in tax noncompliance. In contrast, tax noncompliance is more prevalent in both Turkey and Brazil, with average tax noncompliance rates exceeding 30 percent. However, the size-dependency of tax noncompliance is much lower in Turkey, similar to Spain's level, while in Brazil, a one log-point increase in establishment size leads to a 4 percentage point decrease in tax noncompliance. Angola presents an extreme case with both high tax noncompliance levels and significant size-dependency. The average tax noncompliance rate in Angola, on the other hand, exceeds 50 percent, and the size coefficient is -5.5, indicating a substantial negative relationship between size and tax noncompliance.

I construct comparison economies by calibrating the probability of inspection parameters, η and ρ, to match two key data points in each country's dataset: the average tax noncompliance rate and the size coefficient of tax noncompliance.[25] Additionally, I adjust the output tax rate, τ, for each comparison economy to align the tax revenue-output ratio with that of the benchmark economy.[26]

[25] To calculate the size coefficient in the model, I estimate the same regression as Equation 1 from the model using a model-generated artificial sample. The size coefficient estmates for other countries in the sample can be found in the last column of Table A1.

[26] This neutralization is motivated by the observation that the total tax revenue-to-GDP ratio in the United States and other countries does not exhibit systematic differences. However, the tax collection

Table 5: Cross-country Comparison

	Benchmark (U.S.)	Spain	Turkey	Brazil	Angola
Average Tax Noncompliance (%)	0.0	3.7	37.8	32.9	51.7
Size Coeff. Of Tax Noncompliance (β_1)	0.0	-0.7	-0.9	-4.0	-5.5
Output	100.0	99.8	98.7	97.0	93.7
Average Managerial Quality	100.0	96.7	88.4	77.4	63.0
Mean Size	15.7	15.3	14.4	12.8	10.8
Earning Growth (40-44/25-29) (%)	16.5	15.1	9.9	5.3	0.2
Emp. Share of Large Estb. (%)	45.0	43.4	38.8	29.0	14.7

Notes: This table provides a comparison between the full enforcement benchmark economy and various economies with incomplete tax enforcement: Spain, Turkey, Brazil, and Angola. The first two rows shows the average tax noncompliance rate and the size coefficients of tax noncompliance, estimated using Equation 1. For definitions of the remaining variables, please refer to the explanations under Table 4.

It's important to emphasize that these comparison economies are not fully calibrated versions of their respective real economies. Instead, they represent hypothetical economies in which all model parameters are the same as the benchmark economy, except those governing tax noncompliance moments. In essence, transitioning from the benchmark economy to these hypothetical economies helps answer the question: What would the U.S. economy look like if subjected to the tax enforcement policies of other countries?

Table 5 shows the outcomes of this comparative analysis. Initially, transitioning from the U.S.-calibrated benchmark to Spain, a country characterized by low tax noncompliance and minimal size-dependency, doesn't significantly alter the economic landscape, resulting in only a 3% decline in managerial quality and a slight reduction in output. However, as we venture into countries with more pervasive tax noncompliance, such as Turkey and Brazil, the impact becomes more pronounced. Output experiences a decline of 1.3% and 3% in Turkey and Brazil, respectively. These reductions in output are primarily attributed to substantial drops in average managerial quality, which plummet by 11.6% and 22.6%, respectively, in these countries. Shifting from the benchmark U.S. economy to Angola, a country characterized by an exceptionally high level of tax noncompliance and significant size-dependency, underscores how drastic the potential magnitude of these effects can be. In this transition, we observe a substantial 6.3% decline in output and a striking 37% reduction in managerial quality.

These effects are indeed substantial. To put this into perspective, consider that to gener-

capacities of governments in these countries may significantly vary.

ate a 3% output loss in the benchmark economy, which is equivalent to transitioning from the benchmark to Brazil, an approximate 2.4% reduction in aggregate productivity is needed. In simpler terms, moving from the tax noncompliance-free benchmark economy to an economy like Brazil with incomplete tax enforcement results in an output loss that is akin to reducing aggregate productivity by approximately 2.4% in the benchmark economy.

Additionally, when transitioning to economies with greater size-dependency in tax enforcement, there is a notable shift in the firm size distribution. This shift manifests as a reduced share of employment allocated to large establishments, which, in turn, leads to a decrease in the mean establishment size within the economy. In simpler terms, the prevalence of size-dependent tax enforcement affects the composition of firms and contributes to a smaller average establishment size in the economy.

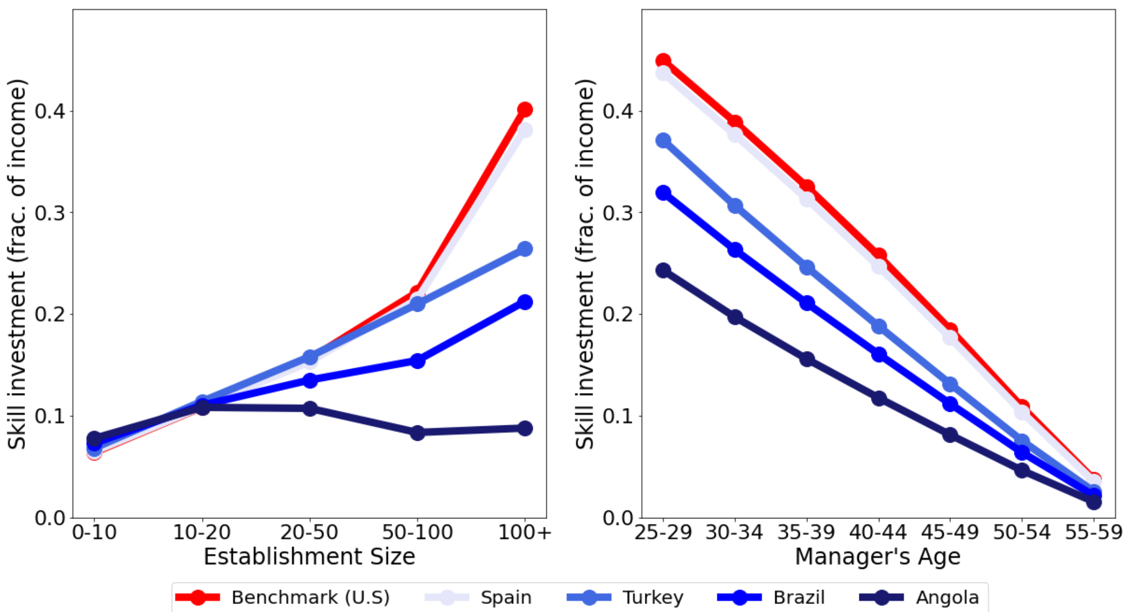
Furthermore, moving towards more size-dependent economies also has a pronounced impact on income growth profiles. Specifically, age-earning profiles become notably flatter, erasing nearly all income growth, particularly evident when transitioning to a country like Angola. This flattening effect signifies that individuals experience much less income growth as they progress from ages 20-25 to 40-45 in these size-dependent tax enforcement economies.

7 Discussion

Comparing the tax noncompliance-free benchmark economy with various incomplete tax enforcement economies provides valuable insights into the magnitude of distortions induced by size-dependent tax noncompliance. Do these distortions affect all managers uniformly, irrespective of the size of their establishments? How do managers of varying age groups respond to incomplete tax enforcement? What is the relative significance of various mechanisms in determining the distortionary effects of size-dependent tax noncompliance? This section aims to address these crucial questions.

Establishment Size and Skill Formation The left panel of Figure 10 presents a visual representation of how the allocation of income for skill investment varies across establishment sizes in both the benchmark economy and other countries. The main insight derived from this comparison is the significant impact of incomplete tax enforcement on the skill investment decisions of managers in large establishments. In contrast, the decisions made by managers in small to mid-sized establishments remain relatively consistent across different economies. In the benchmark economy, managers overseeing establishments with more than 100 workers allocate an average of 40% of their income toward skill formation. However, this percentage decreases as tax noncompliance becomes more dependent on establishment size, dropping to as low as 10% in extreme cases like Angola. This observation can be attributed to the impact of size-dependent tax noncompliance, which exerts a greater distortion on large establishments. In other words, large establishments tend to reduce their factor demands to a relatively larger extent compared to small establishments. Consequently, the return on skill investment for highly skilled managers overseeing large establishments decreases more significantly in comparison to low-skilled managers in charge of small establishments.

Figure 10: Managers' Age, Establishment Size and Skill Formation



Notes: The left panel displays fraction of income invested in managerial skills by establishment size for the economies presented in Table 5. The x-axis is the establishment size categories. For example, '0-10' on the x-axis represents establishments employing between 0 and 10 workers. The right panel illustrates fraction of income invested in managerial skills by the age of managers. The x-axis in this panel represents the ages of managers. For instance, '25-29' on the x-axis corresponds to managers aged between 25 and 29.

Age and Skill Formation The life cycle model of managerial skill formation suggests that the advantage of acquiring an additional unit of managerial skill diminishes over one's lifetime. In this model, the benefit of skill acquisition is represented as the discounted value of the extra profit generated by this additional skill until the end of one's working life. Consequently, as managers age, the benefit of accumulating additional skills naturally decreases. In the benchmark economy, managers in the 25-29 age group allocate approximately 45% of their income to skill formation, but this proportion gradually decreases for older managers, reaching around 2% for managers aged 55-59. The right panel of Figure 10 illustrates the proportion of income allocated to skill investment across different age groups of managers. In economies affected by distortions, managers across all age groups tend to invest fewer resources in skill formation compared to the undistorted benchmark economy. Moreover, the decline in the share of resources dedicated to skill formation is particularly steep among younger managers as the size-dependency of tax noncompliance intensifies.

Decomposition Incomplete tax enforcement affects managerial quality and output through three main channels. The first channel operates through individuals' choices of occupation. The prospect of noncompliance with tax regulations makes the role of a manager more appealing, leading some low-ability individuals who would have otherwise chosen to be workers in a fully enforced tax scenario to opt for managerial positions. This selection of low-ability individuals into managerial roles diminishes the average quality of managers, and we will refer to this as the *occupation channel*. As discussed in the model section, another channel arises from managers' decisions regarding skill investment. The size-dependent nature of tax noncompliance creates a disincentive for managers to invest in their skills, which we will label as the *skill investment channel*. The third channel operates through general equilibrium effects. Incomplete tax enforcement alters individuals' decisions regarding asset holdings due to shifts in disposable incomes and the relative profitability of asset ownership. Furthermore, changes in the skill composition of the economy impact managers' demands for factors of production, while changes in the composition of occupations influence labor supply. To clear both markets, factor prices adjust in equilibrium, and we will refer to this as the *general equilibrium channel* (or simply *prices*).

To gain insights into the relative importance of these channels, I conducted a decomposition analysis, the results of which are summarized in Table 6. Each entry in the table represents a percentage change relative to the benchmark economy. For ease of comparison, Column (1) illustrates the change relative to the benchmark when all channels are operational. Subsequently, in columns 2 to 4, I deactivate each channel one by one by imposing benchmark policy functions or prices. In Column 2, for instance, the skill investment policy function remains the same as in the benchmark, while occupation decisions and prices can be adjusted in equilibrium. Column 3 follows a similar process but deactivates the occupation channel by imposing the benchmark occupation policy function. Finally, Column 4 represents a partial equilibrium scenario in which prices are held constant at benchmark levels.

Column (2) sheds light on the role of the skill investment channel in driving changes in output. When skill investment decisions are not distorted by size-dependent tax enforce-

Table 6: Decomposition

		(1)	(2)	(3)	(4)
	Fixed Channels	-	Skill Inv.	Occupation	Prices
	Operating Channels	All Channels	Occupation + Prices	Skill Inv. + Prices	Occupation + Skill Inv.
Spain	Managerial Quality	-3.3	-2.5	-0.8	-3.6
	Output	-0.2	-0.0	-0.2	-1.0
Turkiye	Managerial Quality	-11.6	-4.8	-4.6	-10.3
	Output	-1.3	-0.1	-1.3	-5.2
Brazil	Managerial Quality	-22.6	-13.7	-8.7	-21.9
	Output	-3.0	-0.7	-2.7	-9.2
Angola	Managerial Quality	-37.0	-25.3	-13.5	-32.0
	Output	-6.3	-0.4	-6.2	-17.8

Notes: This table presents a comparison of the relative importance of three channels operating in the model: *occupation channel*, *skill investment channel*, and *general equilibrium channel* (or simply prices). Each column's first and second entries display the fixed and operational channels. The rest of the entries shows the percentage change relative to the benchmark economy in related variable.

ment, the overall output remains largely unchanged or experiences only slight variations in all countries. However, it's important to note that overall managerial quality drops significantly in these scenarios. Thus, unlike the occupation channel, the skill investment channel plays a pivotal role in influencing output changes. Moving to Column (3), we see that the skill investment channel also holds importance in determining shifts in managerial quality, albeit to a lesser extent compared to the occupation channel. For instance, in the case of Brazil, managerial quality experiences an 8.7 percent decline even when the occupation channel is not operational, emphasizing the role of the skill investment channel in shaping managerial quality. In the last column, which represents a partial equilibrium setup, we find that price adjustments in equilibrium do not significantly impact changes in managerial quality. However, they do have a mitigating effect on output. In simpler terms, the adjustments in prices within the general equilibrium context help alleviate the decline in aggregate output resulting from the deterioration in managerial quality. For example, in Brazil, output experiences a 9.2 percent drop in the partial equilibrium scenario, which is nearly three times higher than the general equilibrium decline of only 3 percent.

8 Concluding Remarks

In this paper, I investigate the relationship between tax noncompliance rates and establishment size, documenting that tax noncompliance tends to be size-dependent and that the degree of size-dependency decreases as GDP per capita rises. To explore the potential implications of these findings on managerial quality and aggregate output, I introduce a modified version of Lucas (1978) span of control model. In this model, managers not only invest in their managerial skills but also decide how much of their sales to report to the government, taking into account the risk of tax inspections. The size-dependent nature of tax noncompliance plays a pivotal role in this model. Through quantitative exercises, I demonstrate that tax noncompliance has a significant negative impact on managerial quality and output. Moreover, the magnitude of this effect is positively correlated with the level of size-dependency in tax noncompliance. For instance, transitioning from a hypothetical tax noncompliance-free U.S. economy to a tax enforcement regime akin to that of Brazil, calibrated to Brazilian data, results in approximately a 23% reduction in average managerial quality and a 3% decrease in output. Additionally, my findings reveal that the distortions caused by size-dependent tax enforcement are more pronounced among younger managers and larger establishments.

The model explored in this paper establishes a negative correlation between output and several key metrics, including managerial quality, mean establishment size, relative earning growth of managers, and the share of managers. In line with these findings, numerous studies have consistently observed a common trend in lower-income countries characterized by lower-quality managers, smaller business establishments, flatter age-earning profiles among managers, and a higher proportion of self-employed individuals.[27] These implications of the model, which are consistent with the data, are solely based on the size-dependent nature of tax noncompliance documented in the data section and underscore the significant macroeconomic consequences.

[27] For the managerial quality, see Bloom et al. (2012) and Bloom and Reenen (2011). For smaller establishments see Gabler and Poschke (2013) and Bento and Restuccia (2017). For the age-earning profiles of managers, see Guner et al. (2018). For the share of self-employment, see Poschke (2019)

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Appendix Data

Tables

Table A1: List of Countries

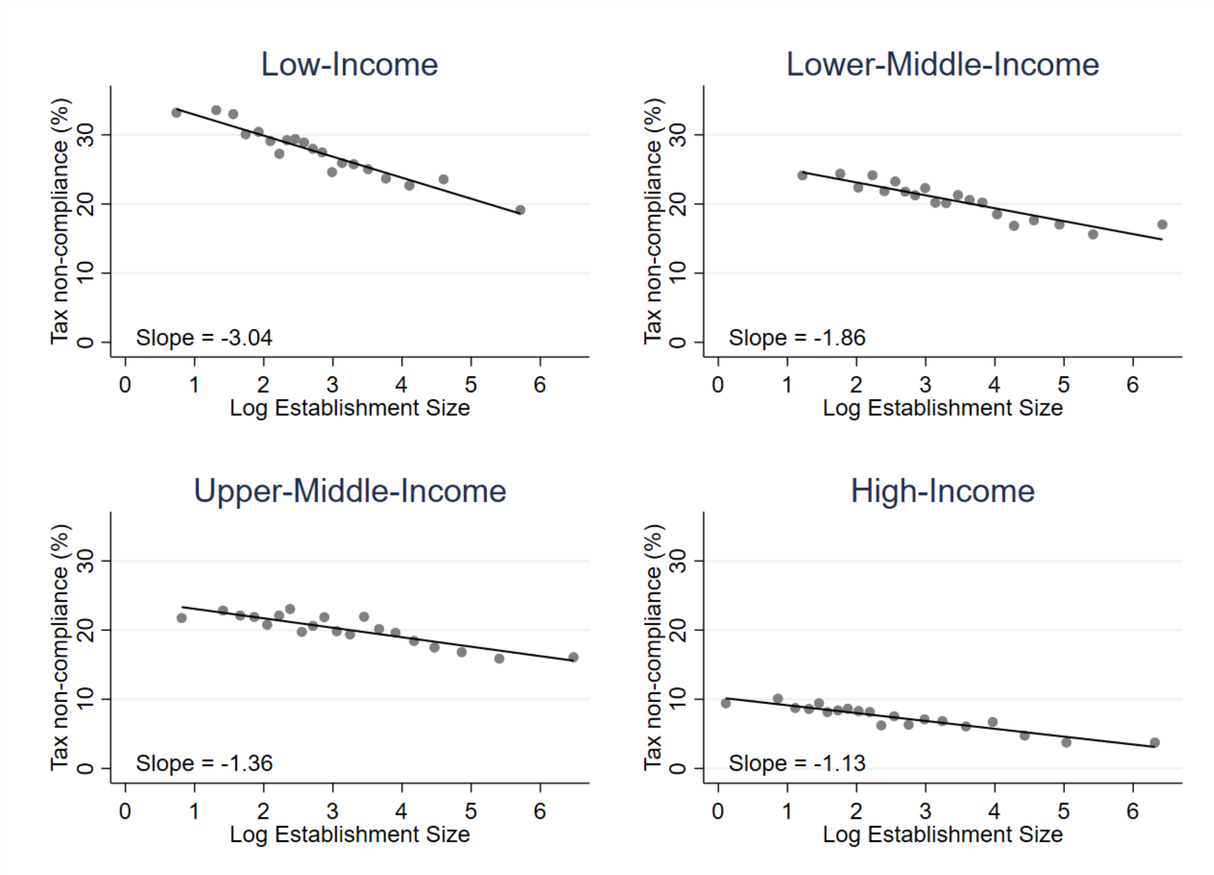
Country	Observations	Years	Average Tax nonCompliance	Size Coefficient (β_1)
Albania	322	2002, 2005	24.9	-1.56*
Algeria	168	2002	27.3	-1.52
Angola	413	2006	51.0	-5.5*
Argentina	943	2006	17.4	-1.97***
Armenia	457	2002, 2005	5.9	-1.3***
Azerbaijan	427	2002, 2005	14.9	-1.31
Belarus	478	2002, 2005	8.4	-1.66***
Benin	169	2004	14.3	-2.51
Bolivia	546	2006	20.3	-4.95***
Bosnia and Herzegov.	280	2002, 2005	22.1	-0.88
Botswana	329	2006	47.2	0.78
Brazil	1,508	2003	32.8	-3.96***
Bulgaria	742	2002, 2004, 2005	13.9	-3.21***
Burkina Faso	130	2006	22.1	-5.08**
Burundi	266	2006	15.2	1.19
Cabo Verde	90	2006	11.4	-8.06**
Cambodia	423	2003	52.0	-4.31***
Cameroon	162	2006	11.9	-2.85***
Chile	1,812	2004, 2006	8.2	-0.4
China	567	2002	39.6	-0.77
Colombia	915	2006	17.1	-2.75***
Congo, Dem. Rep.	339	2006	37.6	-7.09***
Costa Rica	285	2005	28.6	-3.22***
Croatia	297	2002, 2005	9.5	-0.5
Czech Republic	509	2002, 2005	12.8	-1.73***
Dominican Republic	100	2005	39.0	-3.48
Ecuador	970	2003, 2006	24.3	-1.37
Egypt	940	2004	16.8	-1.97***
El Salvador	972	2003, 2006	20.9	-1.58**
Estonia	271	2002, 2005	5.1	-0.57*
Eswatini	288	2006	58.0	-0.74
Gambia	166	2006	67.2	-9.82***
Georgia	278	2002, 2005	25.7	-2.24**
Germany	1,128	2005	6.0	-0.96***
Greece	500	2005	11.1	-1.54***
Guatemala	942	2003, 2006	25.0	-1.47**
Guinea	211	2006	64.4	-4.69
Guyana	155	2004	26.3	-2.81
Honduras	724	2003, 2006	23.6	-1.63**
Hungary	760	2002, 2005	11.5	-1.65***
India	3,802	2006	27.2	-1.67***
Indonesia	696	2003	27.1	-1.61*
Ireland	488	2005	3.9	-0.69***
Jamaica	70	2005	12.3	0.32
Jordan	412	2006	12.2	1.48

Kazakhstan	706	2002, 2005	9.6	-0.3
Kenya	225	2003	14.7	-1
Korea, Rep.	575	2005	10.0	-2.31***
Kyrgyz Republic	374	2002, 2003, 2005	20.9	-0.47
Latvia	299	2002, 2005	10.3	-0.74
Lebanon	292	2006	34.4	0.75
Lesotho	43	2003	15.9	3.74
Lithuania	300	2002, 2005	13.2	-0.35
Madagascar	268	2005	6.3	-1.24**
Malawi	130	2005	30.8	-6.02***
Mali	129	2003	24.3	1.62
Mauritania	228	2006	47.0	-3.92
Mauritius	168	2005	12.6	0.75
Mexico	1,299	2006	23.7	-3.89***
Moldova	528	2002, 2003, 2005	17.6	-2.37***
Mongolia	155	2006	37.2	0.9
Namibia	321	2006	25.5	-0.65
Nicaragua	779	2003, 2006	37.6	-6.74***
Niger	108	2005	13.1	-8.58***
North Macedonia	315	2002, 2005	30.9	-4.94***
Oman	280	2003	29.5	-2.19
Panama	548	2006	37.1	1.54
Paraguay	461	2006	19.0	-2.59**
Peru	1,065	2002, 2006	18.4	-0.81
Philippines	597	2003	21.8	-3.96***
Poland	1,391	2002, 2003, 2005	10.5	-1.47***
Portugal	488	2005	8.4	-1.42***
Romania	738	2002, 2005	8.7	-1.23***
Russian Federation	866	2002, 2005	17.8	-1.93***
Rwanda	206	2006	19.3	-4.88**
Senegal	188	2003	79.5	-3.4***
Slovak Republic	282	2002, 2005	8.6	-1.14**
Slovenia	316	2002, 2005	12.2	-1.05
South Africa	564	2003	8.9	-0.06
Spain	597	2005	3.7	-0.71***
Sri Lanka	323	2004	7.9	-3.21***
Tajikistan	377	2002, 2003, 2005	23.9	-1.66*
Tanzania	625	2003, 2006	41.7	-4.24***
Turkey	2,008	2002, 2004, 2005	37.8	-0.91**
Uganda	724	2003, 2006	40.8	-1.81*
Ukraine	875	2002, 2005	13.2	-0.14
Uruguay	368	2006	14.7	-1.49*
Uzbekistan	499	2002, 2003, 2005	5.7	-1.73***
Vietnam	1,155	2005	11.4	-1.42***
West Bank and Gaza	369	2006	12.8	-3.34***
Zambia	147	2002	15.4	-0.15

Notes: This table lists the countries in the final sample together with number of observation, the year(s) that the data is collected, average tax noncompliance rate, level of size-dependency estimates and their significance levels. ($*p < 0.1, **p < 0.05, ***p < 0.01$).

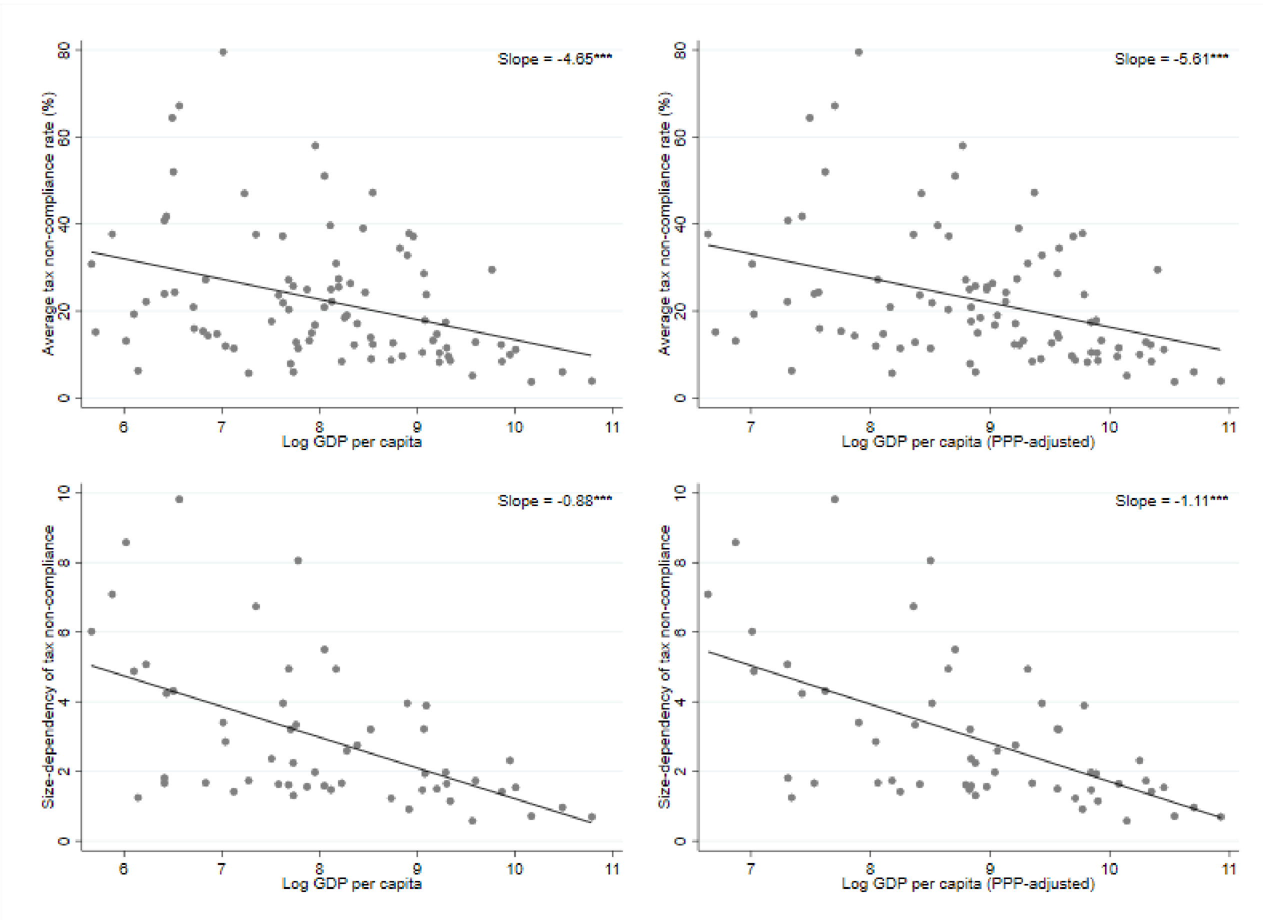
Figures

Figure A1: Tax Noncompliance and Establishment Size



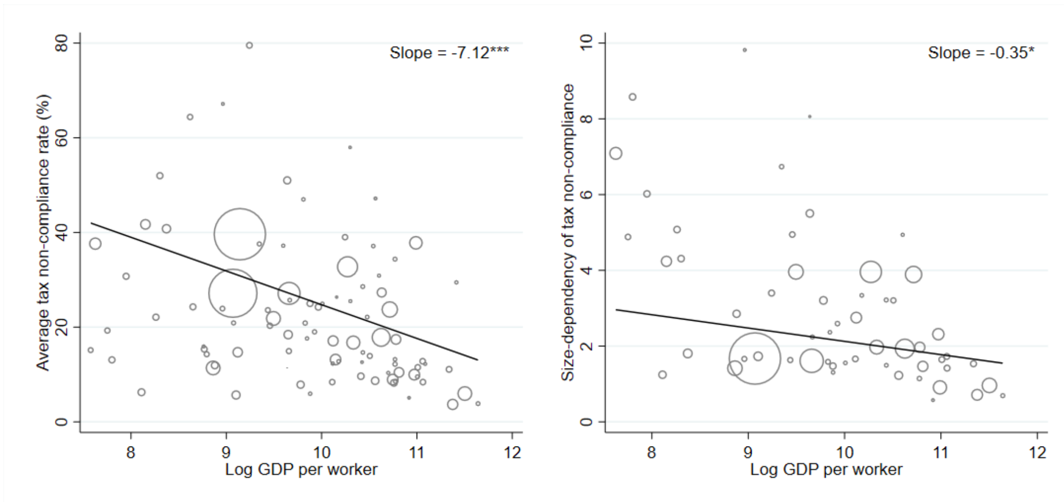
Sources: World Bank Enterprise Surveys. **Notes:** The figure shows binned scatter plots of tax noncompliance rate and log establishment size in the four per capita income groups, by controlling country and year fixed effects, establishments' age, and industry. The binned scatter plots are constructed by the following way: first regressing the y-and x-axis variables on the set of control variables, and generating the residuals from those regressions, second grouping the residualized x-variable into 20 equal-sized bins and computing the mean of the x-variable and y-variable residuals within each bin, finally creating a scatterplot of these 20 data points. *The slope* in each scatter plot is constructed from an OLS regression of the y-residuals on the x-residuals. To categorize countries, I use World Bank Income classification in the year that sample is collected.

Figure A2: Measure of Wealth - Robustness



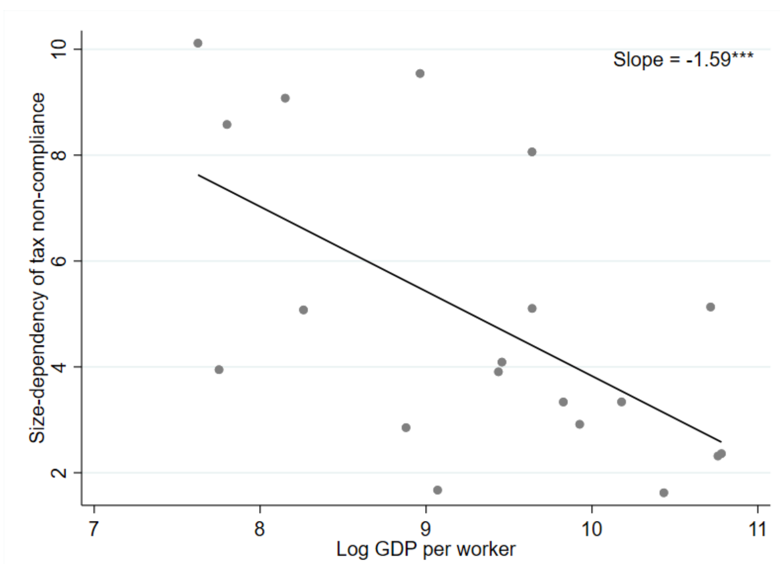
Sources: World Bank Enterprise Surveys, World Bank and author's calculations. **Notes:** The top two panels of this figure are the counterparts of Figure 2 in which log GDP per capita and Purchasing power parity adjusted log GDP per capita are used instead of GDP per worker. Similarly, bottom panels are counterparts of Figure 3. The solid lines are the simple regression line where the size dependency is the dependent variable and the x-axis variable is the independent variable. Corresponding slope coefficients are reported at the upper-right corner of each panel.

Figure A3: Population Weights - Robustness



Sources: World Bank Enterprise Surveys, World Bank and author's calculations. **Notes:** The left panel of this figure shows the population-weighted relationship between the average tax noncompliance rate and the log GDP per worker at the cross-country level. The right panel shows the population-weighted relationship between the level of size-dependency tax noncompliance rate and the log GDP per worker at the cross-country level. Each dot represents a country. The size of the circles is the population of the country.

Figure A4: Sampling Weights - Robustness



Sources: World Bank Enterprise Surveys, World Bank and author's calculations. **Notes:** This figure shows the relationship between the level of size-dependency tax noncompliance rate and the log GDP per worker at the cross-country level for the subsample of countries with survey sampling weights. The solid line is the simple regression line where the size dependency is the dependent variable and the GDP per worker is the independent variable. The slope of the regression line is -1.59 and it is statistically significant at the 1 percent level.

Derivations

The derivative of profit function with respect to managerial ability is the following:

$$\Pi_z(z,r,w) = \Delta[1-\tau+\tau e^*(z)]^{\frac{1}{1-\gamma}} + \Delta[1-\tau+\tau e^*(z)]^{\frac{\gamma}{1-\gamma}} e_z^*(z)z \tag{27}$$

It is sufficient to show that $\Pi_z(z,r,w) > 0$ so that value of being a manager is an increasing function of initial managerial ability z.

$\Pi_z(z,r,w)$ can be reorganized as the following:

$$\Pi_z(z,r,w) = \Delta[1-\tau+\tau e^*(z)]^{\frac{\gamma}{1-\gamma}} [1-\tau+\tau e^*(z) + e_z^*(z)z] \tag{28}$$

where $\Delta > 0$ and $[1-\tau+\tau e^*(z)]^{\frac{\gamma}{1-\gamma}} > 0$. So, we need to show the third expression $1-\tau+\tau e^*(z)+e_z^*(z)z$ is positive, as well.

$$\begin{aligned} 1-\tau+\tau e^*(z)+e_z^*(z)z &= \frac{-\eta}{(\eta z+\rho)^2} + 1 - \tau + \frac{\tau}{\eta z+\rho} \\ &= \frac{-\eta z + (1-\tau)(\eta z+\rho)^2 + \tau(\eta z+\rho)}{(\eta z+\rho)^2} \\ &= \frac{-\eta z(1-\tau) + (1-\tau)(\eta z+\rho)^2 + \tau\rho}{(\eta z+\rho)^2} \\ &= \frac{(1-\tau)(\eta^2 z^2 + \rho^2 + 2\eta z\rho - \eta z) + \tau\rho}{(\eta z+\rho)^2} \\ &= \frac{(1-\tau)(\eta^2 z^2 + \rho^2 + \eta z(2\rho-1)) + \tau\rho}{(\eta z+\rho)^2} \end{aligned} \tag{29}$$

Since $\rho > 1$, $(2\rho-1)$ is positive as well as the entire term $1-\tau+\tau e^*(z)+e_z^*(z)z$. Hence, $\Pi_z(z,r,w) > 0$ for all z.